

October 1<sup>st</sup> 2008

## What's New, What's Possible, What's Coming ...



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Visiting Professor Laureate, University of Newcastle



*"intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication."*



George Polya  
1887-1985

Revised 25/09/2008



## Abstract of Presentation

I will describe and illustrate my experiences over the past two-and-a-bit decades in using, designing and trying to sell mathematical software

I will do this from the perspective(s) of a researcher, an educator, a consultant, and of a partner in a small but robust business

## Outline of Presentation

0. Early **Chronology** of a Company

I. The **Changing Research Landscape**

II. New Ways of **Doing Mathematics**

III. New Ways of **Seeing Mathematics**

IV. Amazing New **Web Services**

### MathResources: Chronology

- 1971: I arrive in Oxford. Meet Borowski 'lexicographically'.
- 1985: Sabbatical in UK/France. We consult for Collins. (And 'stand ready to entertain all invitations'.
- 1995-89: UK Dictionary (first book set from disk). We keep IP and "electronic and musical" rights.  
1991: US Edition. 1993-98: Chinese, Arabic, Italian, Indonesian,...
- 1990: JMB and Carolyn Watters/Jack Ho begin "MathProbe" - hypertext.
- 1993: Technology used for Rio Conference.  
Migration from Pagemaker to Hypercard to SG1 to SUN to Windows (to Mac ?) continues.
- 1994: MRLtd established with Ron Fitzgerald.  
Letter written to HarperCollins.
- 1994-95. Investors and regional funding found.
- 1996: MathResource released (Dec). **Maple inside**
- 1997: MathProbe released (Sept).
- 1998: the future. Constant marketing, distribution, updates, new products.



#### 1998-2002

- Dot bubble burst
- We learned about**
  - Burn rates
  - Naming products
  - Retail software
  - How schools decide
  - Bundling
  - Casio

#### 2002-2008

- \$2.5M from Gov't for Interactive School Math
- Dictionary in Maple
- Reference books die
- Morph in part to contractor: MAA, NIST, NSF, IBM...

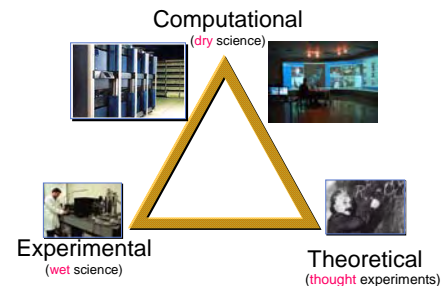
<http://www.mathresources.com/>

## A 1985 Relational Silicon Database



• MacWrite, 3 Apple+'s, a brain dead Lisa, and 5" floppies

## I. Changing Research Landscape: a new triangle

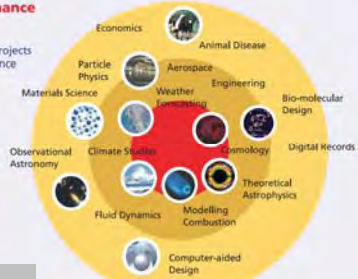


# HPC Needs in Canada or Oz

## 2008 High Performance Computing Needs

The array of Canadian research projects each have unique high performance computing requirements.

- Ring 1 Desktop Computers / Small Clusters
- Ring 2 Mid-Range Systems (in the top 500 worldwide)
- Ring 3 Supercomputers / Terascale Systems (in the top 20 worldwide)



2008: a sustained Petaflop attained at LANL-- 2 years early

The Changing Computing Landscape

Tera becomes Peta becomes ... Bigga

My Lab in Halifax

D-Drive's Nova Scotia location tends us unusual freedom when interacting globally. Many cities around the world are close enough in a chronological sense to comfortably accommodate real-time collaboration.

Dalhousie Distributed Research Institute and Virtual Environment

# Moore's Law and its Implications

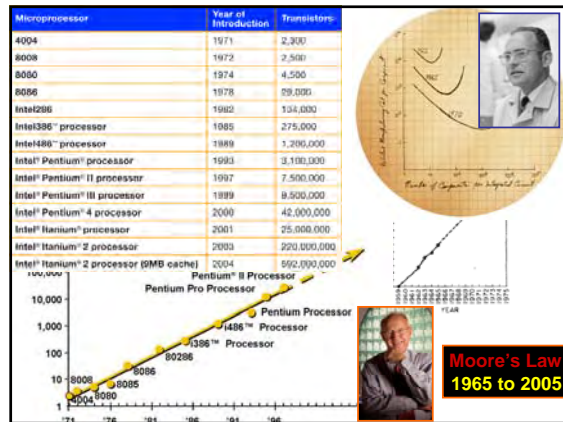
"The complexity for minimum component costs has increased at a rate of roughly a factor of two per year ...

- now taken as "every 18 months to 2 years"

Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years. That means by 1975, the number of components per integrated circuit for minimum cost will be 65,000. I believe that such a large circuit can be built on a single wafer.

Gordon Moore (Intel) "Cramming more components onto [Electronic Circuits](#)", *Electronics Magazine* 19 April 1965

Unprecedented and expected to continue for 10-20 years.



This picture is worth 100,000 ENIACs

The number of ENIACs needed to store the 20Mb TIF the Smithsonian sold me

1947 The past (5Kf/sec)

NERSC's 6000 cpu Seaborg in 2004 (10Tflops/sec) - we need new software paradigms for 'bigga-scale' hardware

The near-present

Mathematical Immersive Reality in Vancouver

## IBM BlueGene/L at LANL

System  
(64 cabinets, 64x32x32)

### IBM Computer Achieves Petaflop Performance

6/9/2008

A National Nuclear Security Administration (NNSA) supercomputer has achieved an operational rate of 1,000 trillion calculations per second, or 1 **petaflop**, making the Roadrunner -- which the NNSA commissioned IBM Corp. to build in 2006 for around \$130 million -- the world's fastest computer, the agency announced today.

2.8/5.6 GF/s  
4 MB

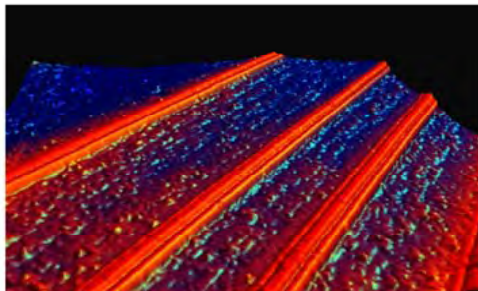
500/12 GF/s  
0.5 GB DDR

217 cpu's: Oct 2007 ran Linpack benchmark at over 596 Tflop/sec (5 x Canada or 8 x Oz)

The future  
2005-2010

## Things we can't model here include:

Self assembling wires 2nm apart (HP Labs)



The Changing Computing Landscape

Simulation by Keuckes-Williams



"It says it's sick of doing things like inventories and payrolls, and it wants to make some breakthroughs in astrophysics."

## II. New Ways of Doing Math

- **and related subjects:** Computer Science, Statistics, Engineering, all Sciences, every other subject .... for learning or for research
  - Experimentally on the Computer
  - Visual or Haptic or Acoustic Output
  - Simulations and Emersions
  - With Web-services, Databases, Wikis, ...
    - Marvelous support tools for the Classroom
- also **New Ways** of **Collaborating**

## 2006 ICM Satellite Meeting Collection

AKPeters, October 2008

"The digital era has dramatically changed the ways that researchers search, produce, publish, and disseminate their scientific work. These processes are still rapidly evolving due to improvements in information science, new achievements in computer science technologies, and initiatives such as DML and open access journals, digitization projects, scientific reference catalogs, and digital repositories.

These changes have prompted many mathematicians to play an active part in the developments of the digital era, and have led mathematicians to promote and discuss new ideas with colleagues from other fields, such as technology developers and publishers. This book is a collection of contributions by key leaders in the field, offering the paradigms and mechanisms for producing, searching, and exploiting scientific and technical scholarship in mathematics in the digital era."



## Jon Borwein's Mathematics Portal

The following is a list of useful math tools. The distinction between categories is somewhat arbitrary.

### Utilities (General)

1. [The On-Line Encyclopedia of Integer Sequences](#)
2. [ISC2.0: The Inverse Symbolic Calculator](#)
3. [3D Function Grapher](#)
4. [Julia and Mandelbrot Set Explorer](#)
5. [The KnotPlot Site](#)

### Utilities (Special)

6. [EZ Face : Evaluation of Euler Sums and Multiple Zeta Values](#)
7. [GraphHedron: Automated and Computer Assisted Conjectures in Graph Theory](#)
8. [Embrece-Trefethen-Wright Pseudospectra and Eigenproblems](#)
9. [Symbolic and Numeric Convex Analysis Tools](#)

### Reference

10. [NIST Digital Library of Mathematical Functions\(X\)](#)
11. [Experimental Mathematics Website](#)
12. [Numbers, Constants, and Computation](#)
13. [Numbers: the Competition](#)
14. [The Prime Pages](#)

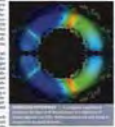
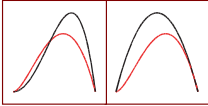


**Experimental Methodology**

- Gaining **insight** and intuition
- Discovering new relationships
- Visualizing** math principles
- Testing and especially **falsifying conjectures**
- Exploring a possible result to see **if it merits formal proof**
- Suggesting approaches for formal proof
- Computing replacing lengthy hand derivations
- Confirming analytically derived results

**MATH LAB**  
Computer experiments are transforming mathematics  
Science News 2004

Mathematical experiments are transforming mathematics. In the past, mathematicians used their brains to discover new results. Now, they use computers to explore mathematical ideas. This approach has led to new discoveries in many areas of mathematics, including number theory, geometry, and physics. For example, computer experiments have helped mathematicians understand the behavior of prime numbers and the structure of fractals. The use of computers also allows mathematicians to test conjectures and explore new ideas in a way that was previously impossible. This new approach to mathematics is called experimental mathematics, and it is changing the way we do math.





Comparing  $-y^2 \ln(y)$  (red) to  $y-y^2$  and  $y^2-y^4$

**"The Crucible"**  
AKPeters November 2008

*The Computer as Crucible*  
An Introduction to Experimental Mathematics

Jonathan Borwein  
Keith Devlin  
with illustrations by Karl H. Ogmann



THE COMPUTER AS CRUCIBLE  
AN INTRODUCTION TO EXPERIMENTAL MATHEMATICS  
JONATHAN BORWEIN • KEITH DEVLIN

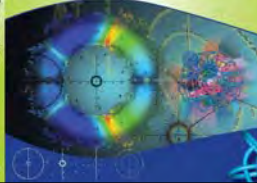
©2008 The MIT Press, Massachusetts

**Experimental Mathematics in Action**

David H. Bailey  
Jonathan M. Borwein  
Neil J. Calkin  
Roland Dreier  
D. Russell Luke  
Victor H. Moll

Much more use of visualization

**Experimental Mathematics in Action**




Experimental Mathematics in Action

A K Peters, Ltd.

**Math + Physics = Computing ?**

- En français



La physique et les mathématiques sont à leur tour égales comparées à l'exécution d'un programme en informatique.

6

©1998 L. L. SODER. N°34-KRISL 2006

**Haptics and Light Paths** D-DRIVE Doug a haptic mascot

Haptic Devices extend the world of I/O into the tangible and tactile

To test latency issues ...





Links multiple devices so two or more users may interact at a distance (BC/NS Demo April 06)

- in Museums, Aware Homes, elsewhere
- Kinesiology, Surgery, Music, Art ...

Sensable's **Phantom Omni**

**Caveman Geometry** (2001)

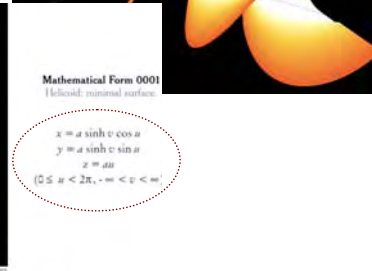
Very cool for the **one** person with control - and very expensive: great genomic applications



## Cost effective 3D visualization in 2007

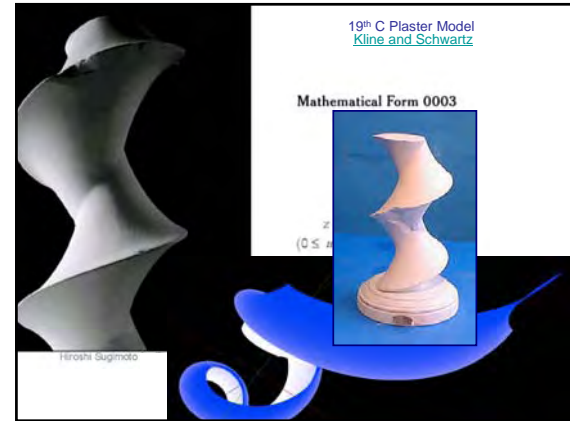


19<sup>th</sup> C model  
plus recent  
photograph and  
21<sup>st</sup> C rendition



Mathematical Form 0001  
Helicoid: minimal surface:

$$\begin{aligned}x &= a \sinh v \cos u \\y &= a \sinh v \sin u \\z &= au \\(0 \leq u < 2\pi, -\infty < v < \infty)\end{aligned}$$



19<sup>th</sup> C Plaster Model  
[Kline and Schwartz](#)

Mathematical Form 0003



Hirosshi Sugimoto



## Coast to Coast ('C2C') Seminar

2008: will focus on  
PhD presentations  
Chile has now joined

Lead partners:

**Dalhousie D-Drive** – Halifax  
Nova Scotia

**IRMACS** – Burnaby,  
British Columbia

Other Participants so far include:

University of British Columbia, University  
of Alberta, University of Alberta, University  
of Saskatchewan, Lethbridge University,  
Acadia University, MUN, Mt Allison, St  
Francis Xavier University, University of  
Western Michigan, MathResources Inc,  
University of North Carolina, ...

Tuesdays 3:30pm (Atlantic) 11:30am (Pacific)

✓Chapter in [Communicating Mathematics in the  
Digital Era](#) (AK Peters, Sept 2008)

I could be in Newcastle AG  
**CARMA** is coming  
Computer Assisted Research Maths  
and its Applications

### The Experience

Fully interactive multi-way audio and video

Given good bandwidth audio is  
much harder (if you rehearse)

The closest thing to being in the same room

Shared Desktop for  
viewing presentations or  
sharing software

## Content Dominates Form



Jonathan Borwein, Dalhousie University  
Mathematical Visualization

High Quality Presentations



Peter Borwein, IRMACS  
The Riemann Hypothesis

Uwe Glaeser, Simon Fraser University  
Semantic Blueprints of Discrete Dynamic Systems

"No one explains chalk!"

Jonathan Schaeffer, University of Alberta  
Solving Checkers



Arvind Gupta, MITACS  
The Protein Folding Problem

Przemyslaw Prusinkiewicz, University of Calgary  
Computational Biology of Plants




Karl Dilcher, Dalhousie University  
Fermat Numbers, Wieferich and Wilson Primes

Future Libraries will include  
very complex objects



## III. New Ways of Seeing Math

- The Colour Calculator**
  - numbers as pictures
- The Inverse Calculator**
  - numbers go in and symbols come out
- The Top Ten Numbers Website**


All at <http://ddrive.cs.dal.ca/~isc/portal>

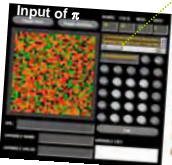
## A Colour and an Inverse Calculator (1995 & 2007)

Aesthetic base for middle-school maths (Nathalie Sinclair)  
 Art:  $228/71 = \pi = 22/7$

### Inverse Symbolic Computation

**Inferring mathematical structure from numerical data**

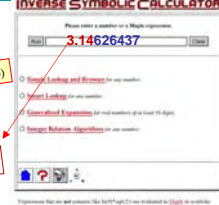
- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs *micro-parallelism*
- It faces the "curse of exponentiality"
- Implemented as **identify** in **Maple 9.5**



Input of  $\pi$

identify( $\sqrt{2} + \sqrt{3}$ )

$\sqrt{2} + \sqrt{3}$

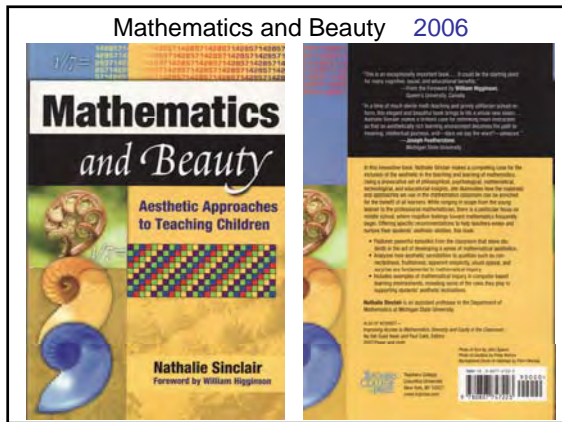


INVERSE SYMBOLIC CALCULATOR

What value is closest to 3.14626437?

3.14626437

Identify  $\sqrt{2} + \sqrt{3}$




Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

Standard lookup results for 12.587886219548403854

$\exp(1) \cdot \pi^2$

ISC The original IC

The Dev Team: Hathan Steger, Andrew Stoddick, Lingyan Ye, Timus Diale, Peter Debrayari, Diana Moroz, O-Yeol Choi, Jon Borwein

3.146264370 [Try it]

19.99909998 [Try it]

ISC The original IC

The Dev Team: Hathan Steger, Andrew Stoddick, Lingyan Ye, Timus Diale, Peter Debrayari, Diana Moroz, O-Yeol Choi, Jon Borwein

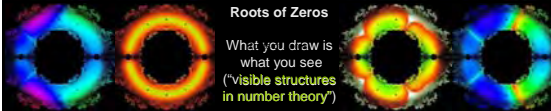
accepts either floating point expressions or correct Maple syntax as input. However, for Maple syntax requiring too long for evaluation, a limited list has been implemented.

Visit:

- [Jon Borwein's Website](#)
- [David Bailey's Website](#)
- [Math Resources Portal](#)

• **ISC+ runs on Gloopcap**

• **Less lookup & more algorithms than 1995**



Roots of Zeros

What you draw is what you see ("visible structures in number theory")

### Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

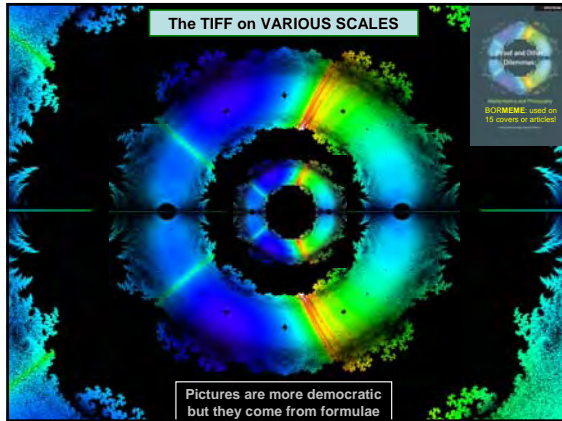
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. **The color scale represents a normalized sensitivity** to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at 3600 dpi)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the  $x^9$  term
- The **white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!"  
 Greg Chaitin, [Interview](#), 2000.





## When is a Movie an Interactive Proof?

### The Perko Pair $10_{161}$ and $10_{162}$

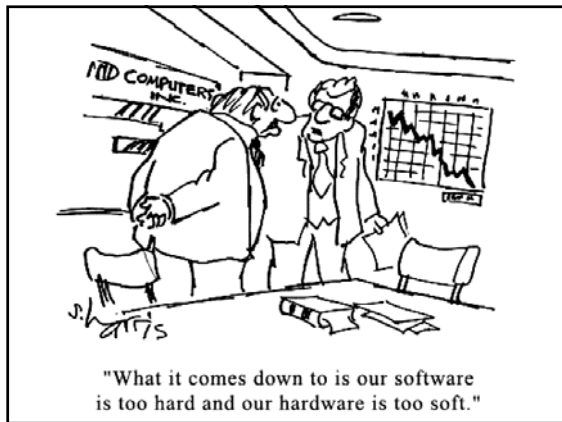
are two adjacent 10-crossing knots (1900)

- first shown to be the same by Ken Perko in 1974
- and beautifully made dynamic in [KnotPlot](#) (open source-ish)

## A Movie that Teaches Beautifully

- [Arnold and Rogness \(2007\)](#)

Möbius Transformations Revealed

$$f(z) = \frac{az + b}{cz + d}$$


## IV. Amazing New Web Services

- [AT&T Online Encyclopedia of Sequences](#)  
What is 1,2,3,6,11,23,47,106,235,...?
- [NIST Digital Library of Math Functions](#)  
What is an Airy Function?
- [MAA Digital Library](#) with my company's [free dictionary](#)  
– also in [Maple](#) since 9.5

Supernumerary Rainbow over Newton's birthplace

MathResources Inc.

PlanetMath.org MathDL Wolfram MathWorld

Search the texts will also do lots of the maths

### Greetings from the On-Line Encyclopedia of Integer Sequences!

Matched Up to a Stack of 30 Found Her 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560 561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860 861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880 881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900 901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920 921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000

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The faint line below the main colored arc is a "supernumerary rainbow", produced by the interference of different sun-rays traversing a raindrop and emerging in the same direction. For each color, the intensity profile across the rainbow is an Airy function. Airy invented his function in 1838 precisely to describe this phenomenon more accurately than Young had done in 1800 when pointing out that supernumerary rainbows require the wave theory of light and are impossible to explain with Newton's picture of light as a stream of independent corpuscles. The house in the picture is Newton's birthplace.



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"The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

• J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.