AustMS Australian Mathematical Society Inc.

Mathematical Visualization and other Learning Tools

Jonathan Borwein, FRSC www.cs.dal.ca/~jborwein
Canada Research Chair in Collaborative Technology

``intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication. ***Therefore, I think that in teaching high school age youngsters we should emphasize intuitive insight more than, and long before, deductive reasoning."***

George Polya

Atlantic Computational Excellence Network

ACEnet

DALHOUSIE UNIVERSITY Inspiring Minds

WestGrid

Revised 15/09/05

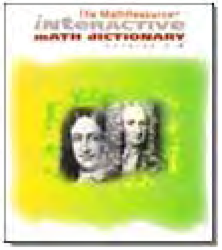
Polya Made Plausible by Computers

"A mathematical deduction appears to Descartes as a chain of conclusions, a sequence of successive steps. What is needed for the validity of deduction is intuitive insight at each step which shows that the conclusion attained by that step evidently flows and necessarily follows from formerly acquired knowledge (acquired directly by intuition or indirectly by previous steps). I think that in teaching high school age youngsters we should emphasize intuitive insight more than, and long before, deductive reasoning."

"This "quasi-experimental" approach to proof can help to de-emphasis a focus on rigor and formality for its own sake, and to instead support the view expressed by Hadamard when he stated "The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it."

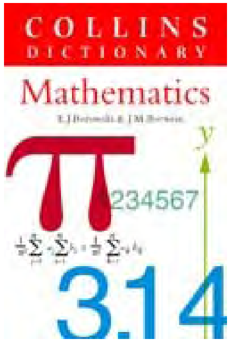
George Polya, ***Mathematical discovery***: *On understanding, learning, and teaching problem solving* (Combined Ed.), New York, Wiley, 1981.

ABSTRACT

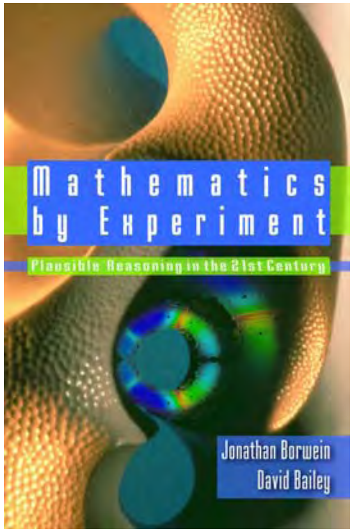


Current and expected advances in mathematical computation and scientific visualization make it possible to display mathematics in many varied and flexible ways. I'll explore some of the present opportunities to integrate graphic and other tools into the curriculum --- for pedagogic and aesthetic reasons.

URLS
http://projects.cs.dal.ca/ddrive/
http://users.cs.dal.ca/~jborwein/
http://www.experimentalmath.info
http://www.mathresources.com

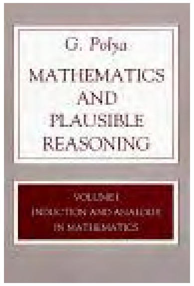


Outline of Presentation



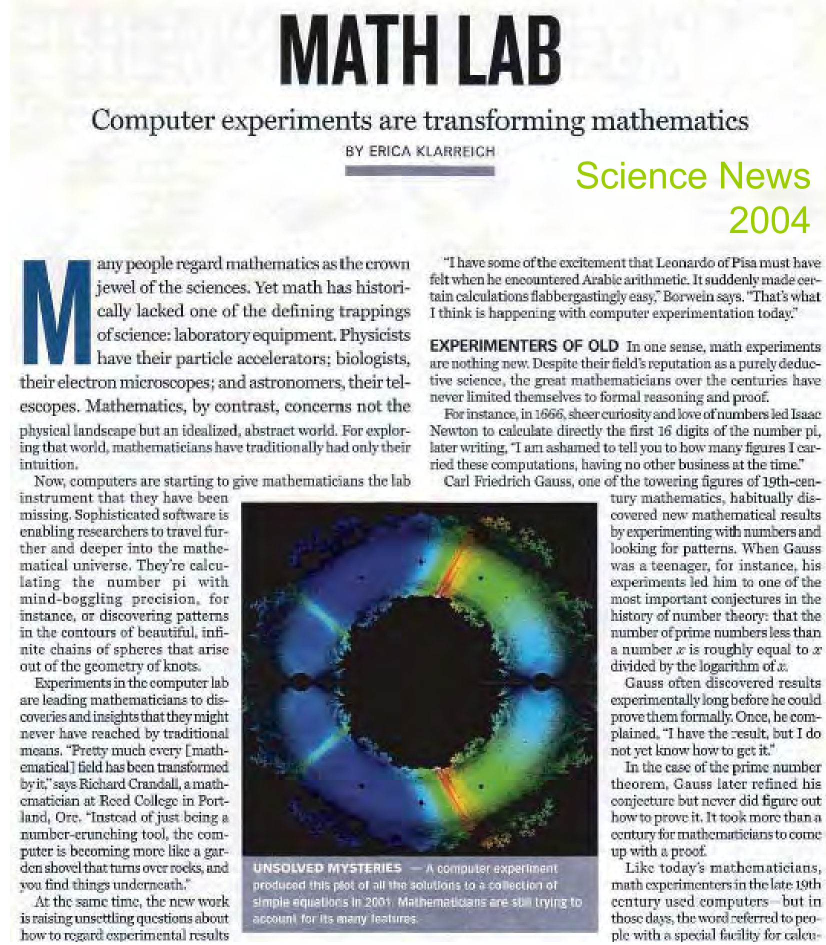
I. Experimental Mathodology
II. Collaborative Tools
III. Visualization and Data Mining
IV. Inverse and Color Calculators
V. Access Grid & Atlantic Gateway to Math
VI. University-Industry Partnerships
   - MathResources Software

References



I. Experimental Mathodology

1. Gaining insight and intuition
2. Discovering new relationships
3. Visualizing math principles
4. Testing and especially falsifying conjectures
5. Exploring a possible result to see if it merits formal proof
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results



MATH LAB

Computer experiments are transforming mathematics

BY ERICA KLARREICH

Many people regard mathematics as the crown jewel of the sciences. Yet math has historically lacked one of the defining trappings of science: laboratory equipment. Physicists have their particle accelerators; biologists, their electron microscopes; and astronomers, their telescopes. Mathematics, by contrast, concerns not the physical landscape but an idealized, abstract world. For exploring that world, mathematicians have traditionally had only their intuition.

Now, computers are starting to give mathematicians the lab instrument that they have been missing. Sophisticated software is enabling researchers to travel further and deeper into the mathematical universe. They're calculating the number pi with mind-boggling precision, for instance, or discovering patterns in the contours of beautiful, infinite chains of spheres that arise out of the geometry of knots.

Experiments in the computer lab are leading mathematicians to discoveries and insights that they might never have reached by traditional means. "Pretty much every [mathematical] field has been transformed by it," says Richard Crandall, a mathematician at Reed College in Portland, Ore. "Instead of just being a number-crunching tool, the computer is becoming more like a garden shovel that turns over rocks, and you find things underneath."

At the same time, the new work is raising unsettling questions about how to regard experimental results

UNSOLVED MYSTERIES — A computer experiment produced this plot of all the solutions to a collection of simple equations in 2001. Mathematicians are still trying to account for its many features.

"I have some of the excitement that Leonardo of Pisa must have felt when he encountered Arabic arithmetic. It suddenly made certain calculations flabbergastingly easy," Borwein says. "That's what I think is happening with computer experimentation today."

EXPERIMENTERS OF OLD In one sense, math experiments are nothing new. Despite their field's reputation as a purely deductive science, the great mathematicians over the centuries have never limited themselves to formal reasoning and proof.

For instance, in 1666, sheer curiosity and love of numbers led Isaac Newton to calculate directly the first 16 digits of the number pi, later writing, "I am ashamed to tell you to how many figures I carried these computations, having no other business at the time."

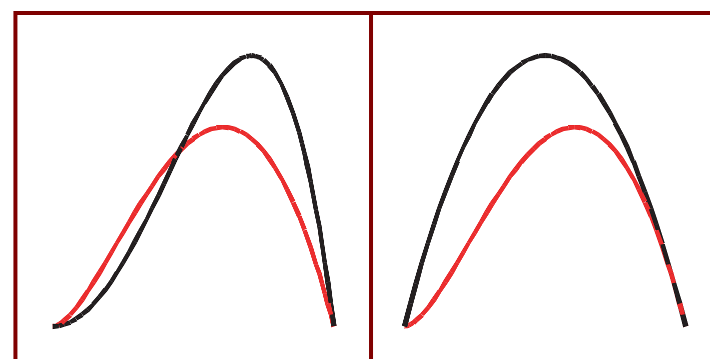
Carl Friedrich Gauss, one of the towering figures of 19th-century mathematics, habitually discovered new mathematical results by experimenting with numbers and looking for patterns. When Gauss was a teenager, for instance, his experiments led him to one of the most important conjectures in the history of number theory: that the number of prime numbers less than a number x is roughly equal to x divided by the logarithm of x.

Gauss often discovered results experimentally long before he could prove them formally. Once, he complained, "I have the result, but I do not yet know how to get it."

In the case of the prime number theorem, Gauss later refined his conjecture but never did figure out how to prove it. It took more than a century for mathematicians to come up with a proof.

Like today's mathematicians, math experimenters in the late 19th century used computers—but in those days, the word referred to people with a special facility for calcu-

Science News 2004



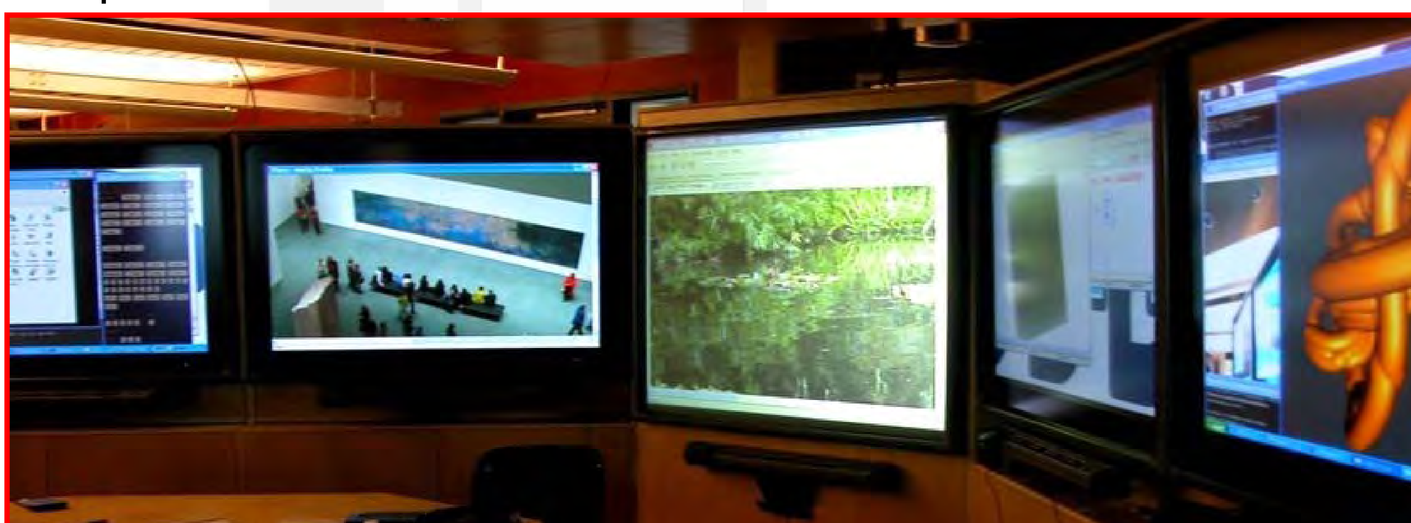
Comparing $-y^2\ln(y)$ (red) to $y-y^2$ and y^2-y^4

"It says it's sick of doing things like inventories and payrolls, and it wants to make some break-throughs in astrophysics."

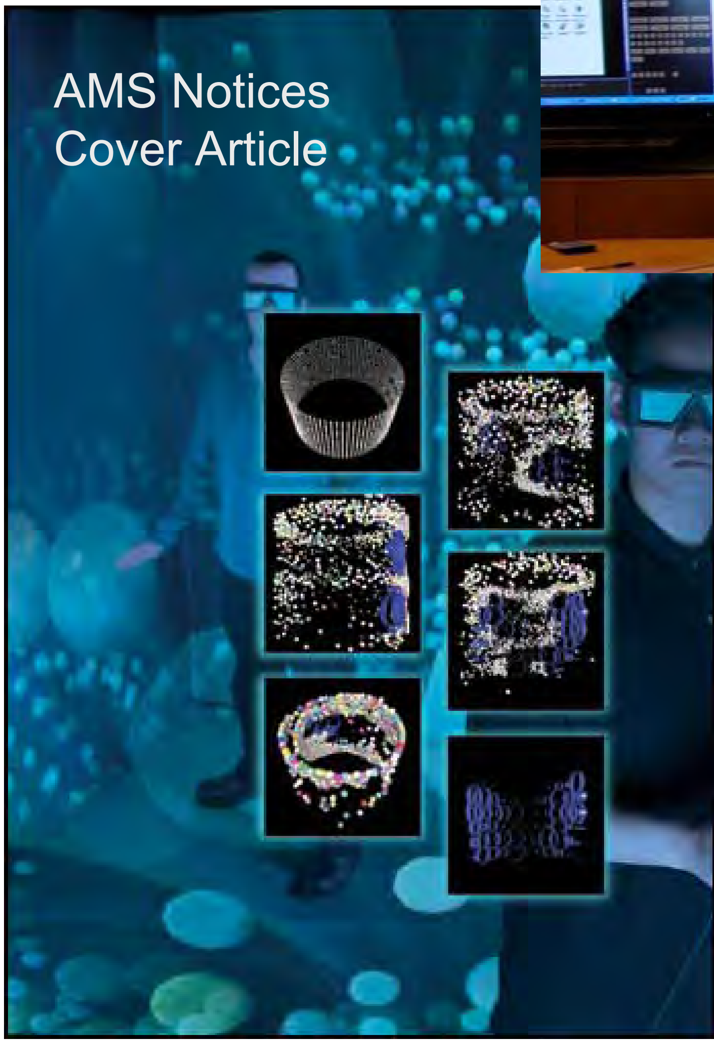
II. Collaboration goes National: East meets West

Welcome to D-DRIVE whose mandate is to study and develop resources specific to distributed research in the sciences with first client groups being the following communities

- High Performance Computing
- Mathematical and Computational Science Research
- Science Outreach
  - ✓ Educational
  - ✓ Research



5 SMART Touch-sensitive Interwoven Screens



Experimental Inductive Mathematics

Our web site:

www.experimentalmath.info

contains all links and references

“Elsewhere Kronecker said ``In mathematics, I recognize true scientific value only in concrete mathematical truths, or to put it more pointedly, only in mathematical formulas." ... I would rather say ``computations" than ``formulas", but my view is essentially the same.”

Harold Edwards, *Essays in Constructive Mathematics*, 2004

Centre seen as 'serious nirvana'

April 07, 2005 , vol. 32, no. 7

By Carol Thorbes

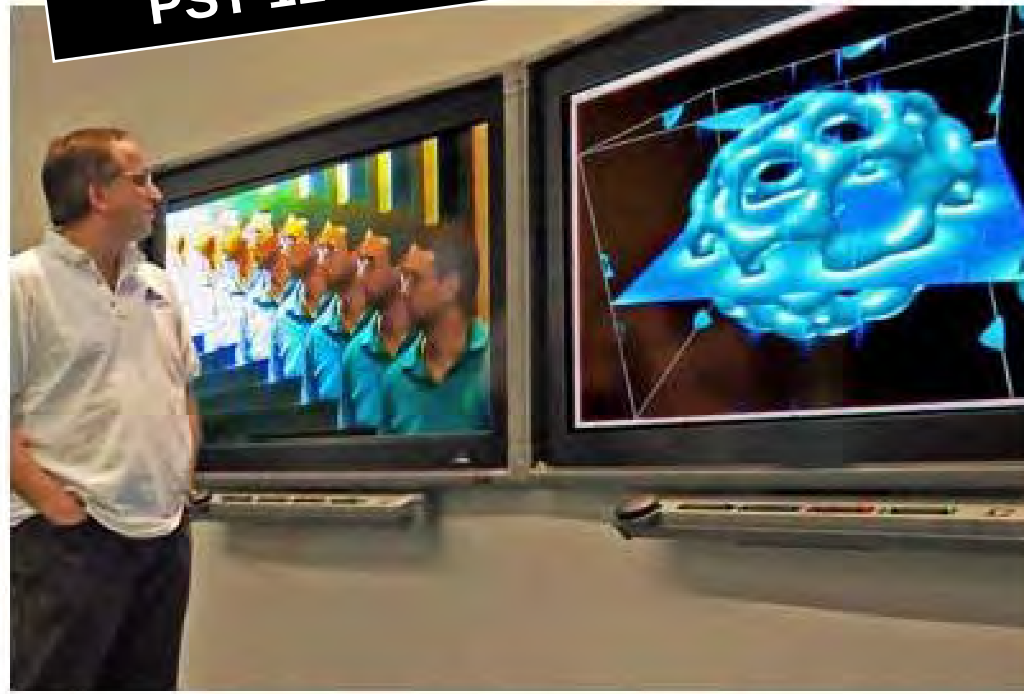
Trans-Canada Seminar Thursdays PST 11.30 MST 12.30 AST 3.30

The 2,500 square metre IRMACS research centre

- ✓The building is a also a 190cpu G5 Grid
- ✓At the official April opening, I gave one of the four presentations from D-DRIVE

Move over creators of Max Head-room, Matrix and Metropolis. What researchers can accomplish at Simon Fraser University's IRMACS centre rivals the high tech feats of the most memorable futuristic films.

The $14 million centre's acronym stands for interdisciplinary research in the mathematical and computational sciences. The centre's expansive view of the ... from atop ... ain echoes its ... al as a facility ... tering ... research ... s whose ... is the computer.



SFU mathematician and IRMACS executive director Peter Borwein (left) communicates with IRMACS collaboration and visualization coordinator Brian Corrie. To the right of them another plasma display portrays a 3D image of a molecular structure.

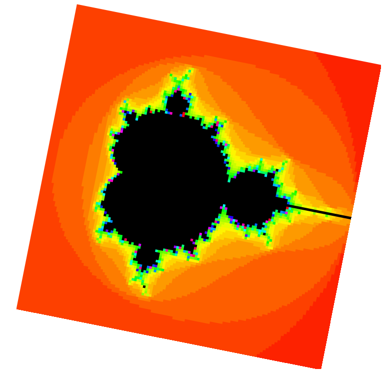
...cted 2,500 square metre space atop the applied sciences building, the centre has eight ...ng rooms and a presentation theatre, seating up to 100 people. They are equipped with ...ble computational, multimedia, internet and remote conferencing (including satellite) technology. High performance distributed computing and clustering technology, designed at SFU, and

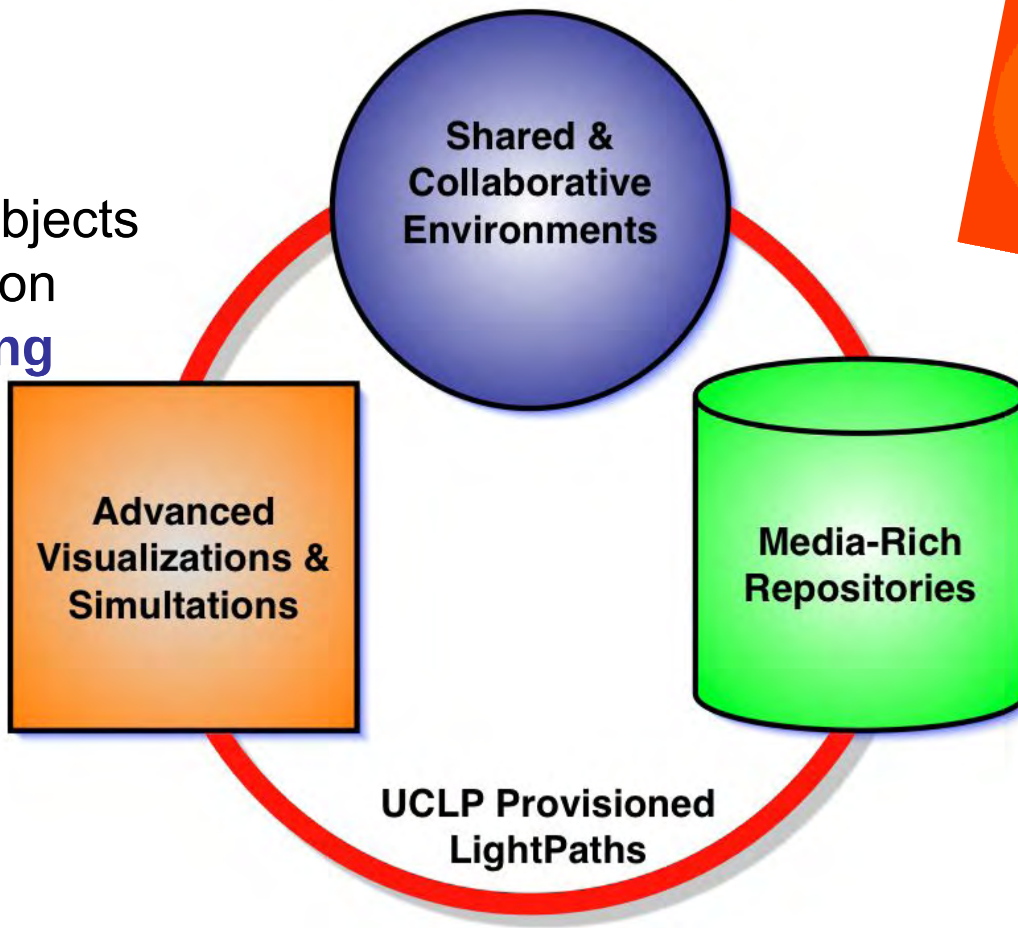

The present
SFU - Colab
Pierre Boulanger
SFU - Colab
University of Alberta
westgrid...
UBC - MAGIC
Netera AG
The AG in Action
in CoLab

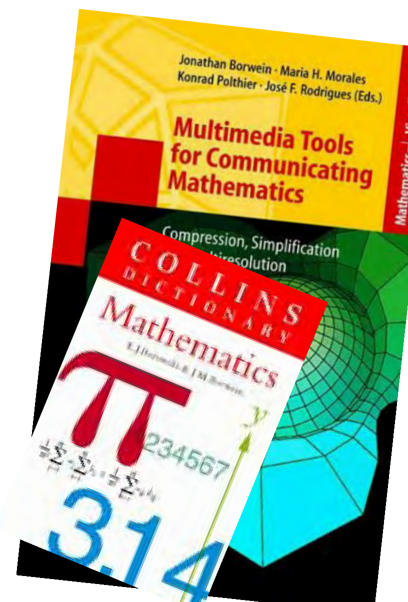
Advanced Networking …

Components include

- **AccessGrid**
- **UCLP** for
  - ✓ haptics
  - ✓ learning objects
  - ✓ visualization
- **Grid Computing**







Haptics in the MLP

Haptic Devices extend the world of I/O into the tangible and tactile

Drive
Dalhousie Distributed Research Institute and Virtual Environment

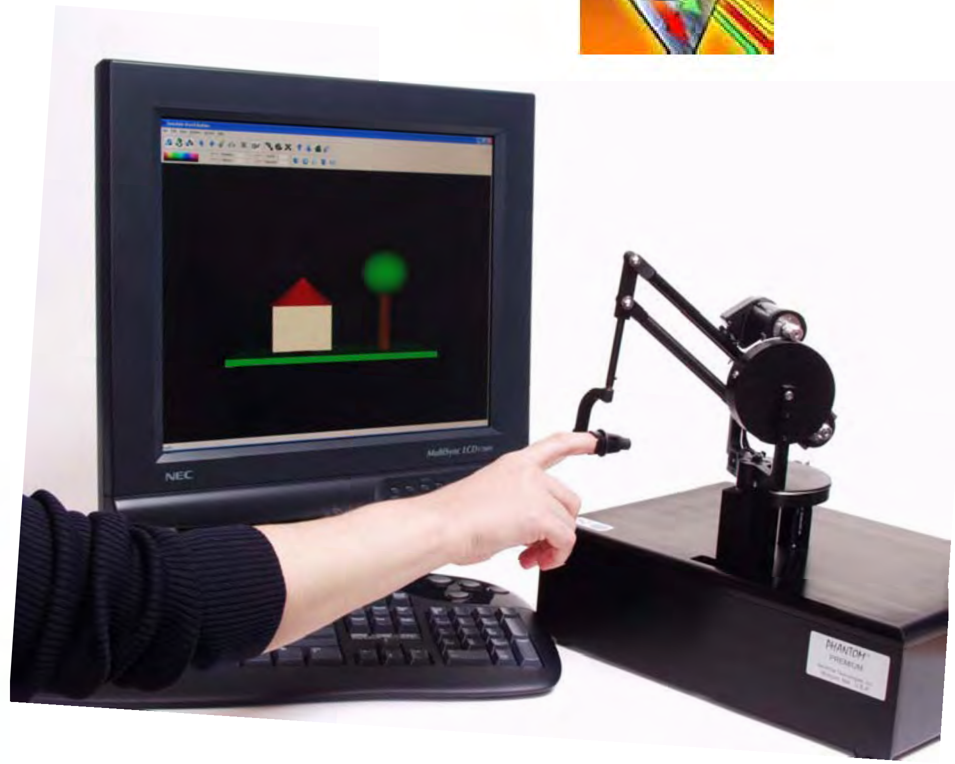
We are linking multiple devices together such that two or more users may interact at a distance

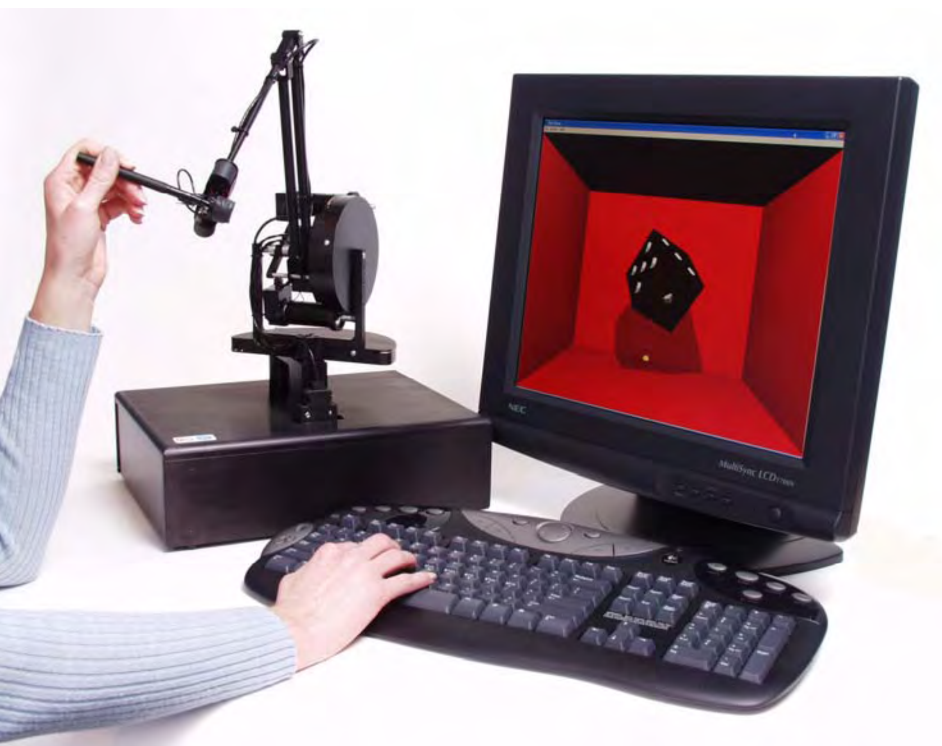
- in Museums and elsewhere
- Kinesiology, HCI ….

Sensable's **Phantom Omni**

And what they do

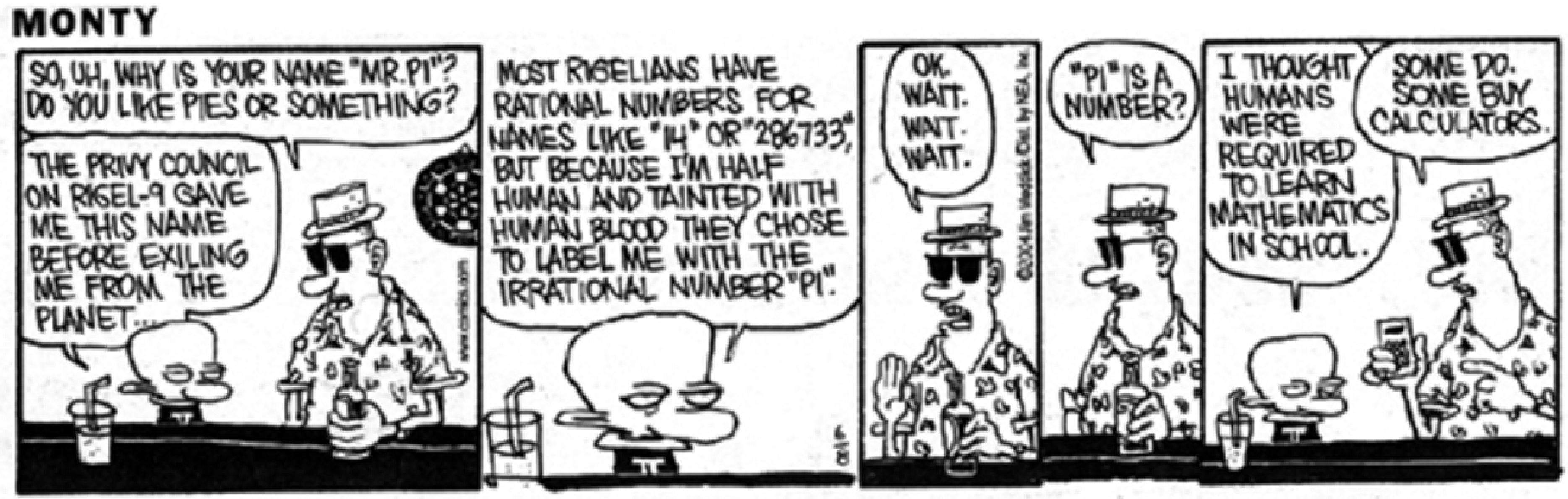
Force feedback informs the user of his virtual environment adding an increased depth to human computer interaction





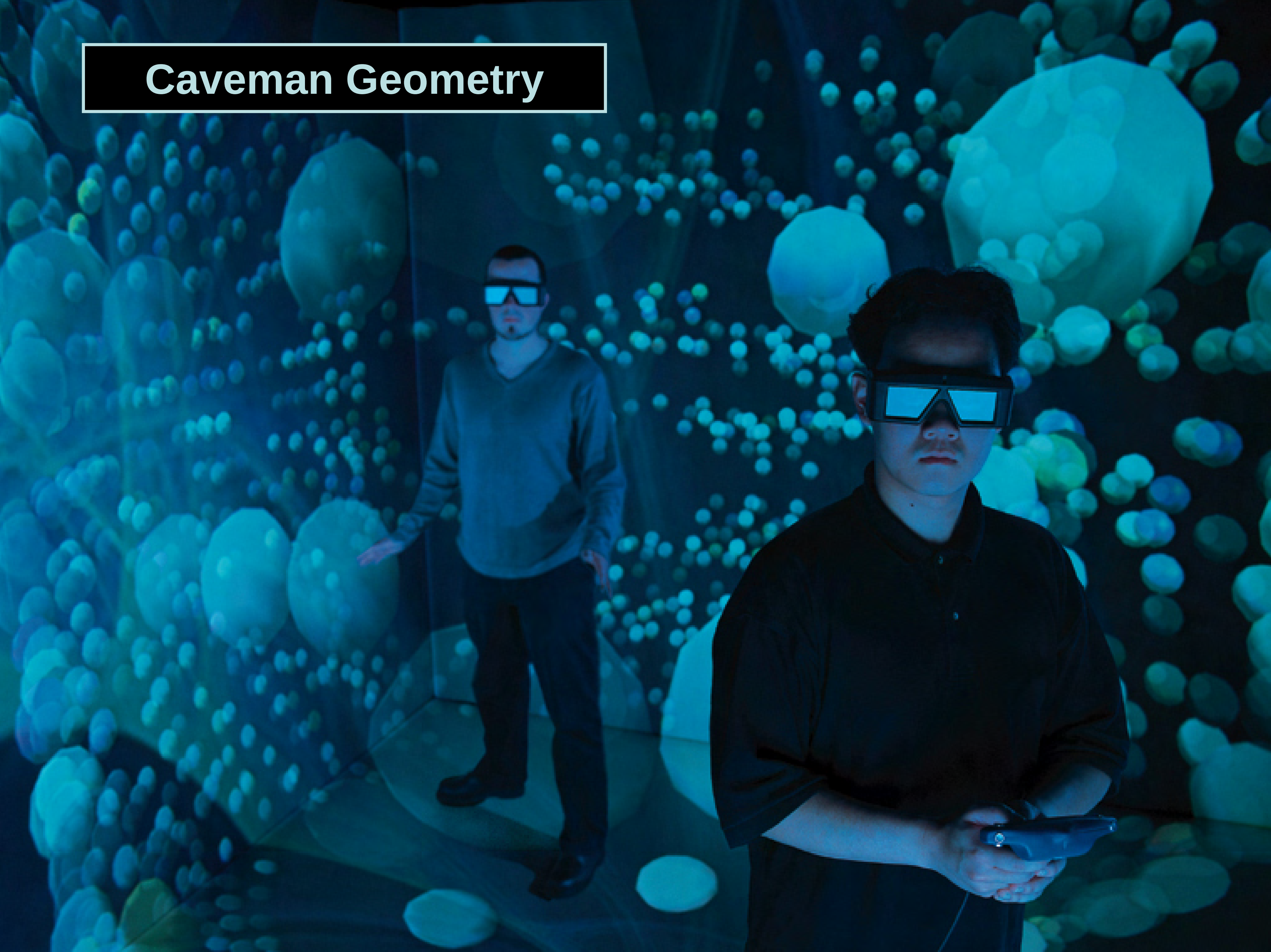
The user feels the contours of the virtual die via resistance from the arm of the device

Mr Pi


MONTY
SO, UH, WHY IS YOUR NAME "MR. PI"? DO YOU LIKE PIES OR SOMETHING?
THE PRIVY COUNCIL ON RIGEL-9 GAVE ME THIS NAME BEFORE EXILING ME FROM THE PLANET...
MOST RIGELIANS HAVE RATIONAL NUMBERS FOR NAMES LIKE "14" OR "286733", BUT BECAUSE I'M HALF HUMAN AND TAINTED WITH HUMAN BLOOD THEY CHOSE TO LABEL ME WITH THE IRRATIONAL NUMBER "PI".
OK. WAIT. WAIT. WAIT.
"PI" IS A NUMBER?
I THOUGHT HUMANS WERE REQUIRED TO LEARN MATHEMATICS IN SCHOOL.
SOME DO. SOME BUY CALCULATORS.

III. Visualization now and in the future

Caveman Geometry



May 2005 AMS Notices Cover

About the Cover

Extreme 3D visualization

The background image of this month's cover is a photograph included by Jonathan Borwein and David Bailey, perhaps somewhat whimsically, in their article on experimental mathematics. The photograph was taken for a publicity brochure for the now defunct New Media Innovation Centre in downtown Vancouver, British Columbia, an organization partially sponsored by Simon Fraser University, to which Borwein is affiliated. The two young men, who are graduate students in the the department of Electrical and Computer Engineering at the University of British Columbia, are in a kind of box with what might be called surround-projection. The approximate spheres are displayed in duplicate at rapidly alternating times in synchronization with the goggles they are wearing, so that what they see is a simulated 3D image, not just the flat projections on the walls on their box. The projections are interactive, controlled by input through a key pad held by Timothy Chen, the student on the right. The project the students are involved in is part of Mr. Chen's student work at U. B. C. What is being projected is a flow field of spheres in a cylinder with various obstacles interactively superimposed into the flow. The inset photographs are screen displays produced by Mr. Chen from the same project.

It's hard to imagine exactly what role such high end visualization technology will play in mathematical research, but not impossible. One likely application for similar, but not quite so sophisticated, display systems might very well be in public presentations. The effects can be spectacular.

Brian Corrie of Simon Fraser University provided us with the digital version of the background photograph.

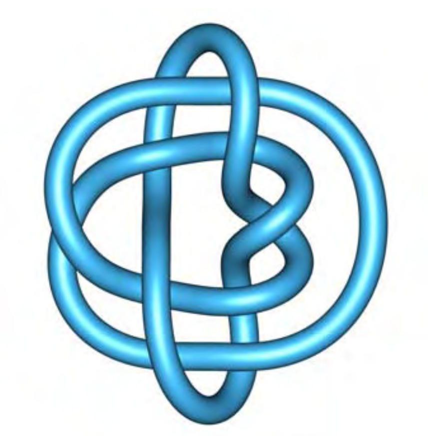
—*Bill Casselman, Graphics Editor*
(notices-cover@ams.org)

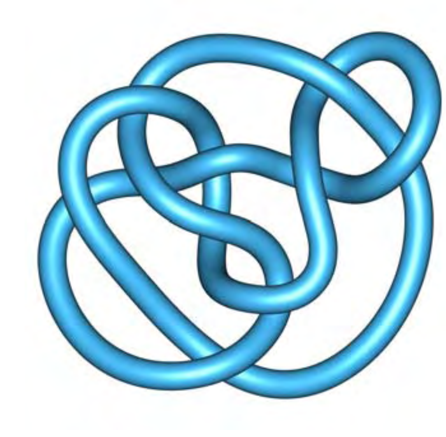
Visualization

Rob Scharein's KnotPlot

The Perko Pair 10_{161} and 10_{162}

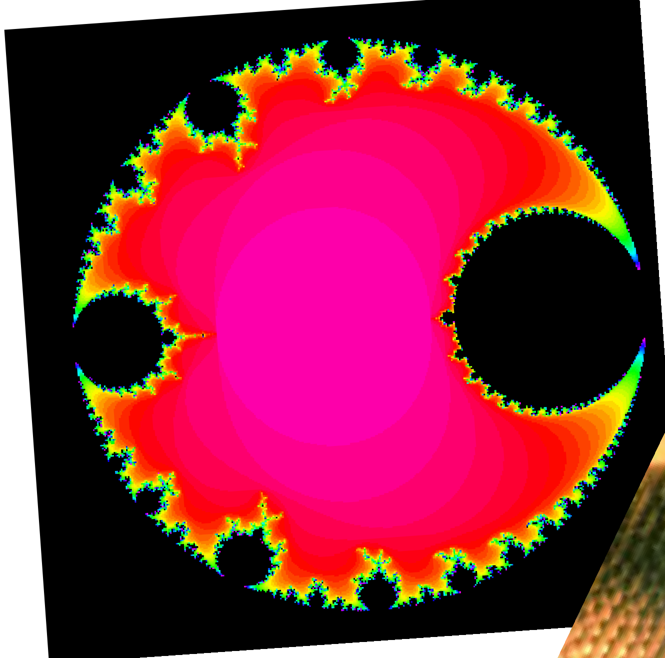
are two adjacent 10-crossing knots (1900)



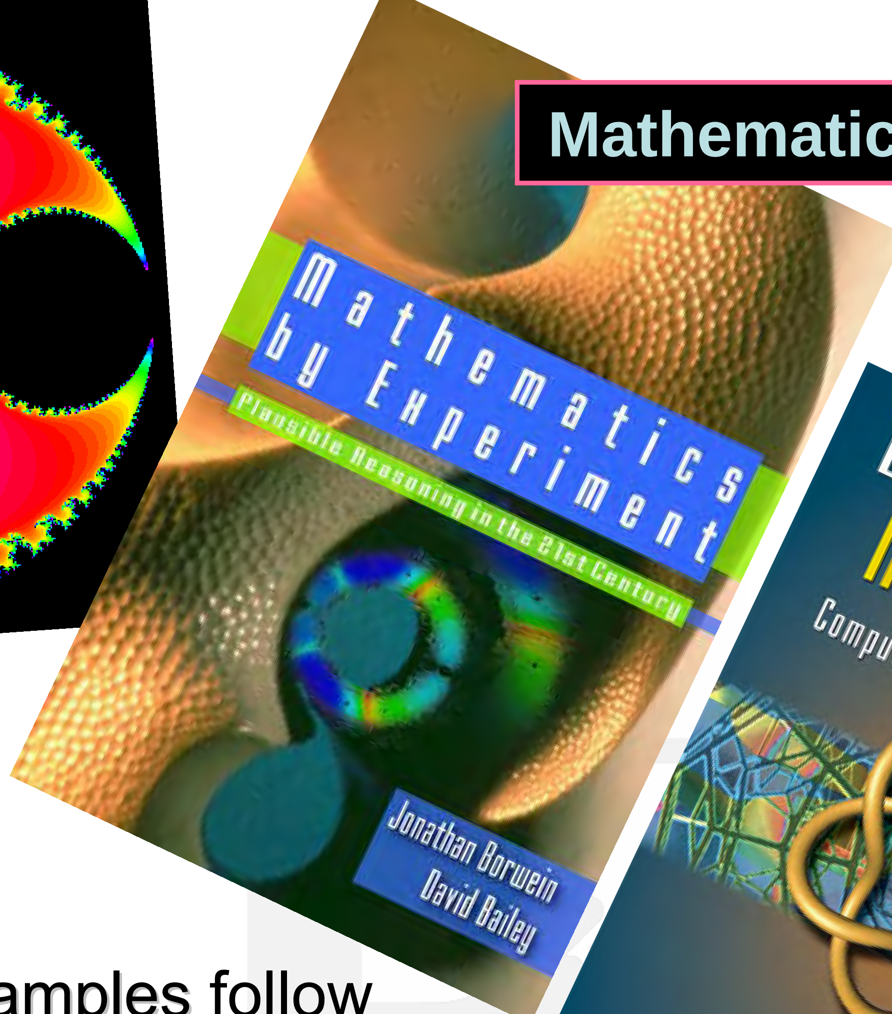


- first shown to be the same by Ken Perko in 1974
- and beautifully made dynamic in KnotPlot

Mathematical Data Mining



An unusual Mandelbrot parameterization

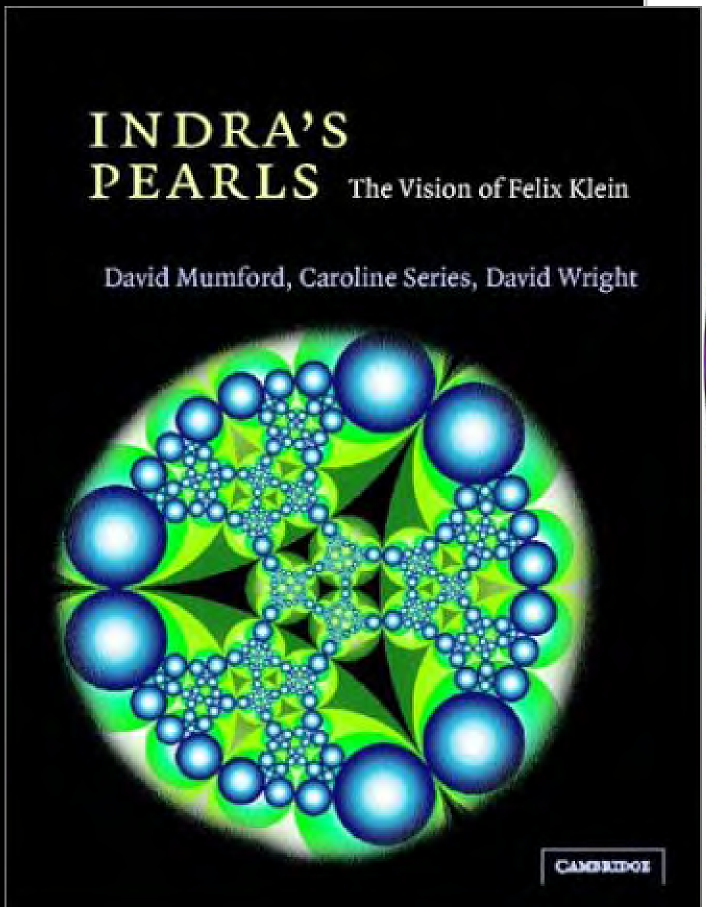


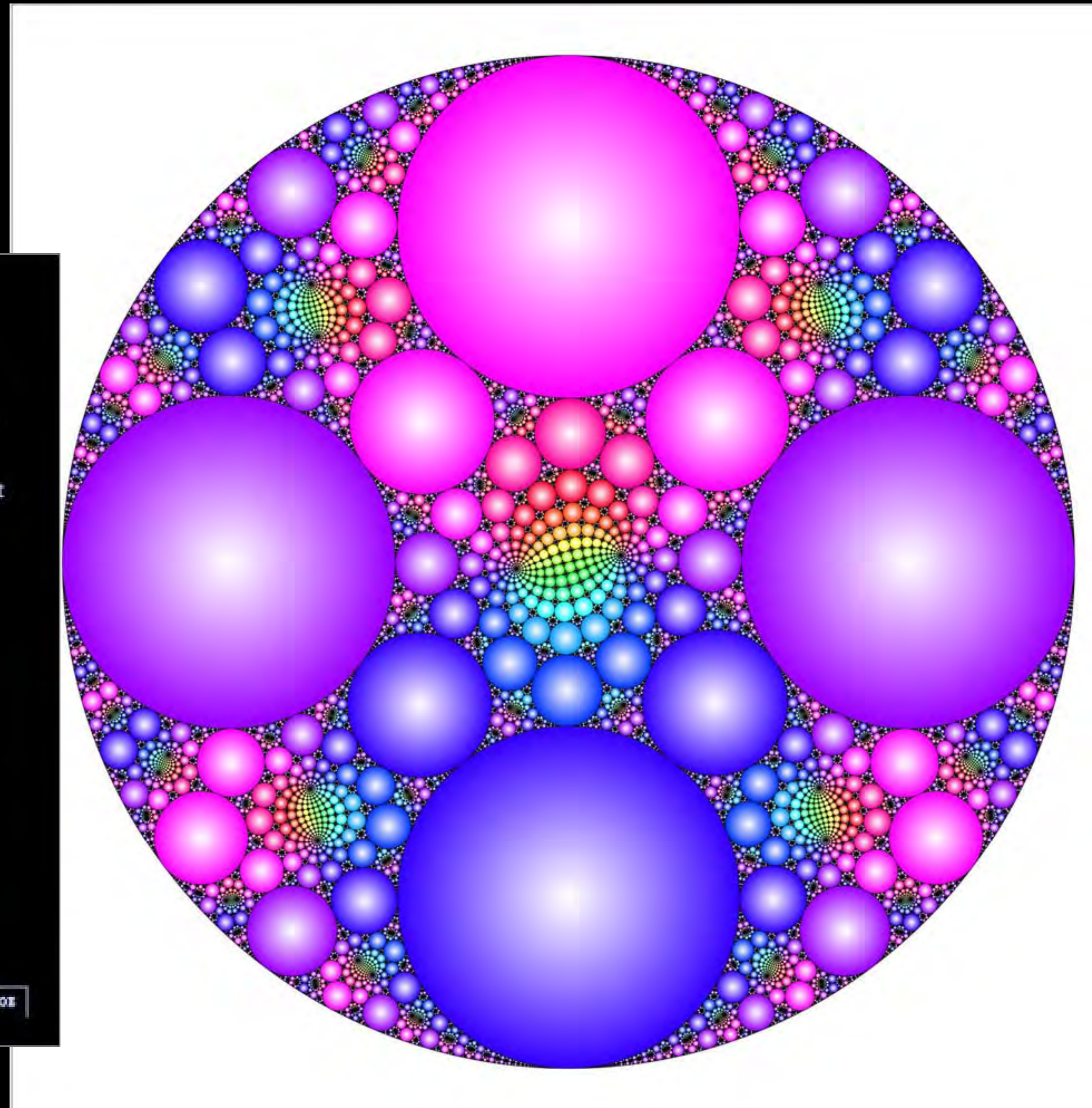
AK Peters, 2004
(CD in press)

Various visual examples follow

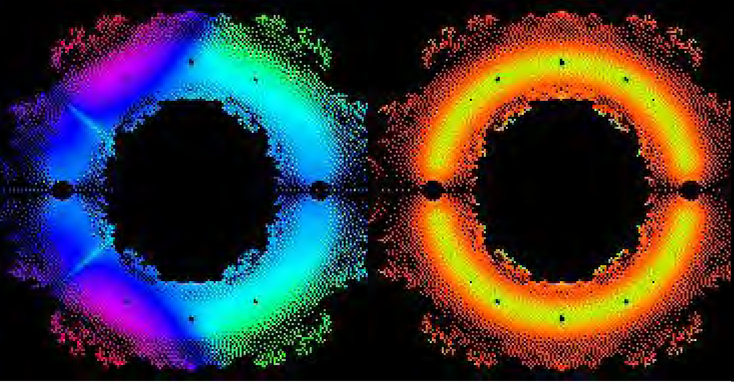
- ✓ Roots of `1/-1' polynomials
- ✓ Ramanujan's fraction
- ✓ Pseudospectra
- ✓ Code optimization

Indra's Pearls
A merging of 19th and 21st Centuries



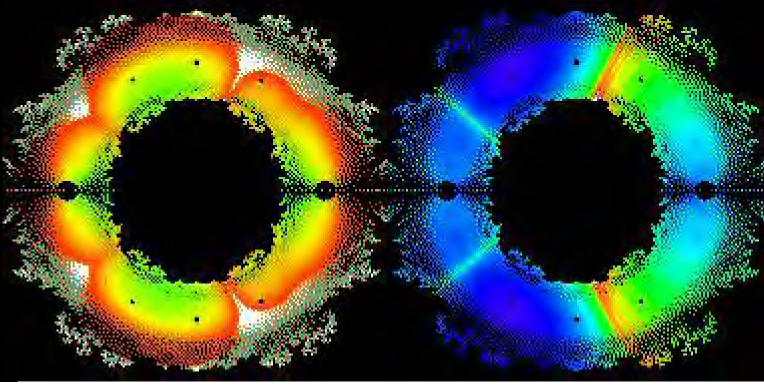


http://klein.math.okstate.edu/IndrasPearls/



Roots of Zeros

What you draw is what you see (visible patterns in number theory)



Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

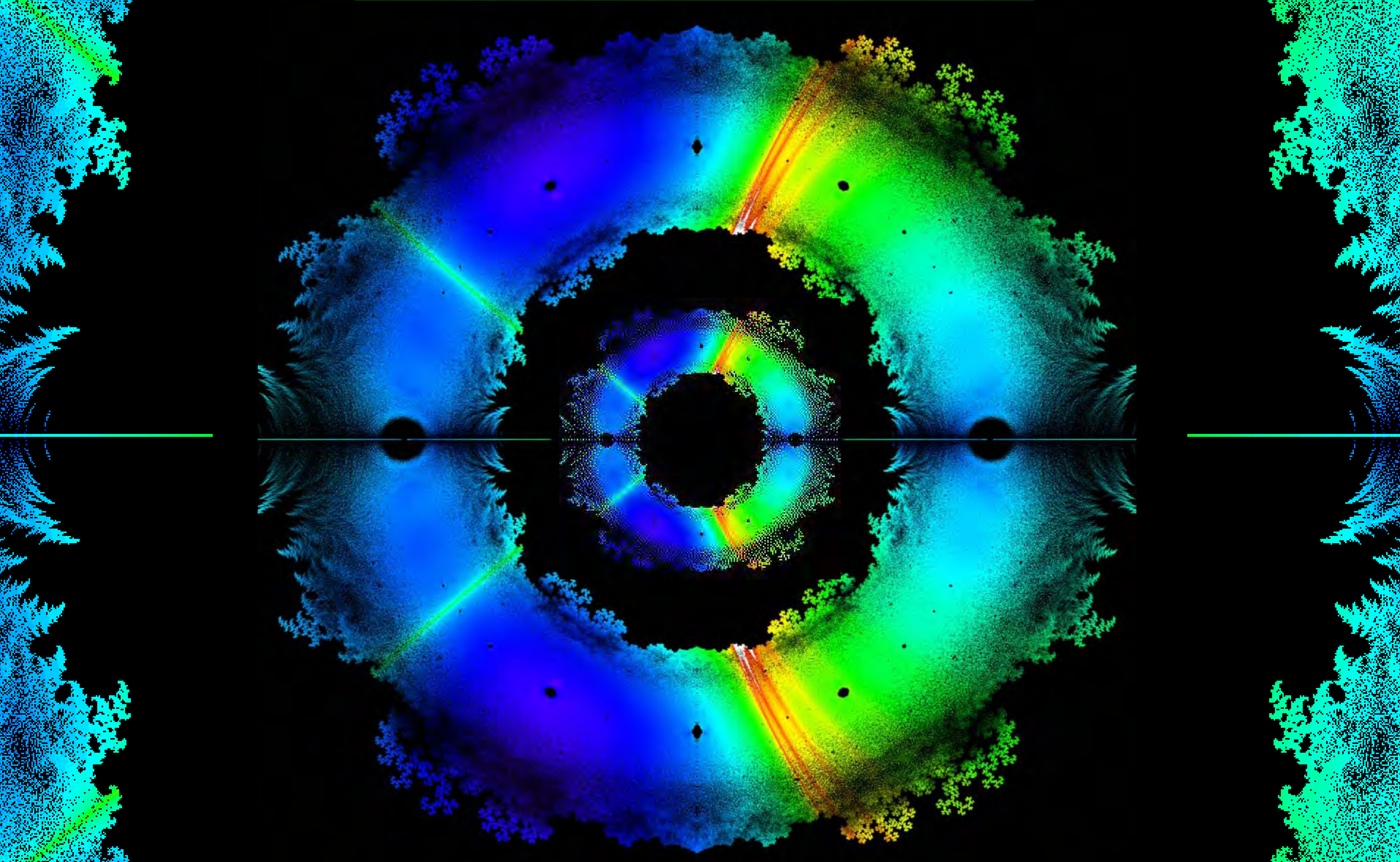
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. The color scale represents a normalized sensitivity to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at **3600 dpi**)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $\mathbf{x^9}$ term
- **The white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

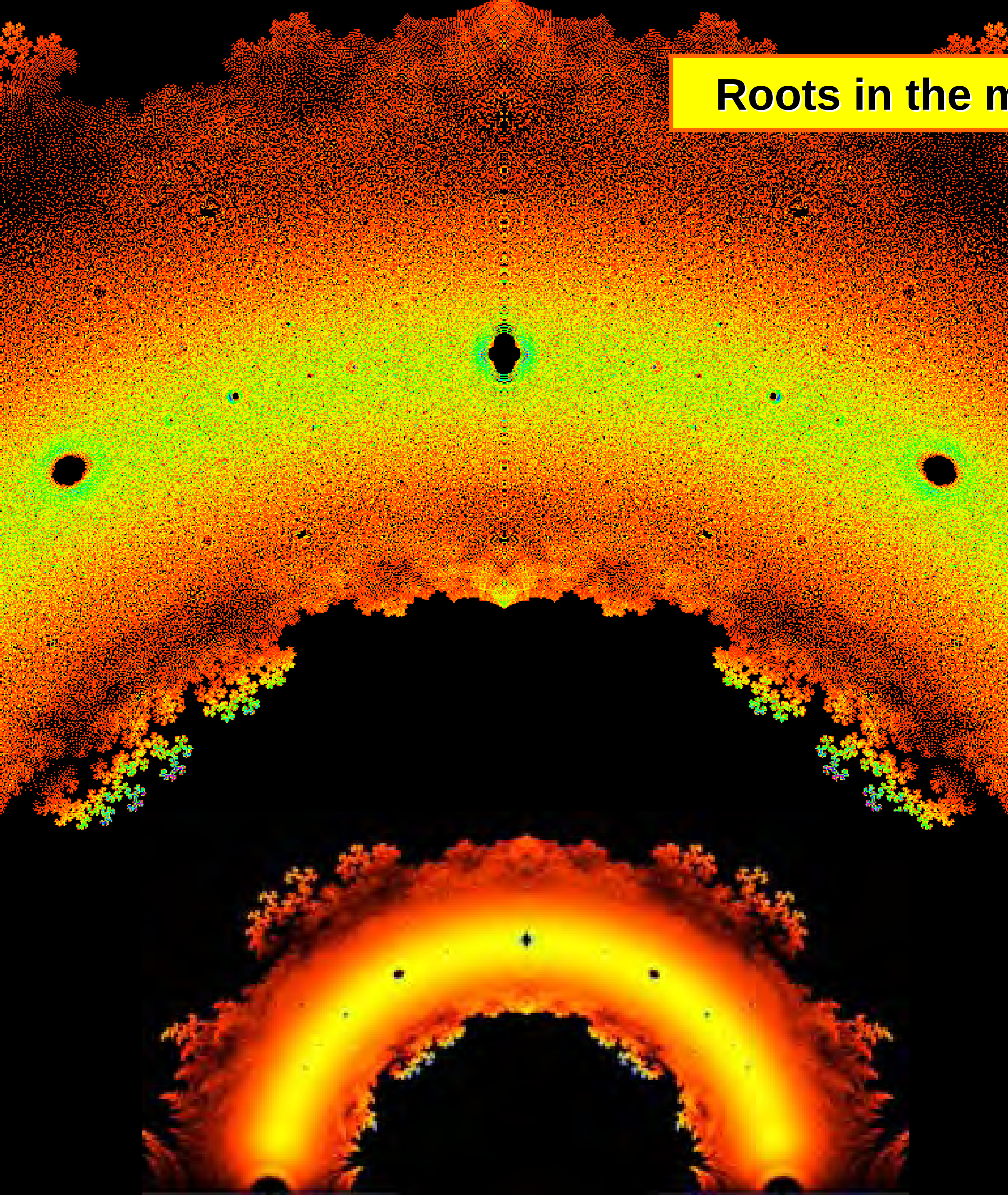
"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!" Greg Chaitin, Interview, 2000.

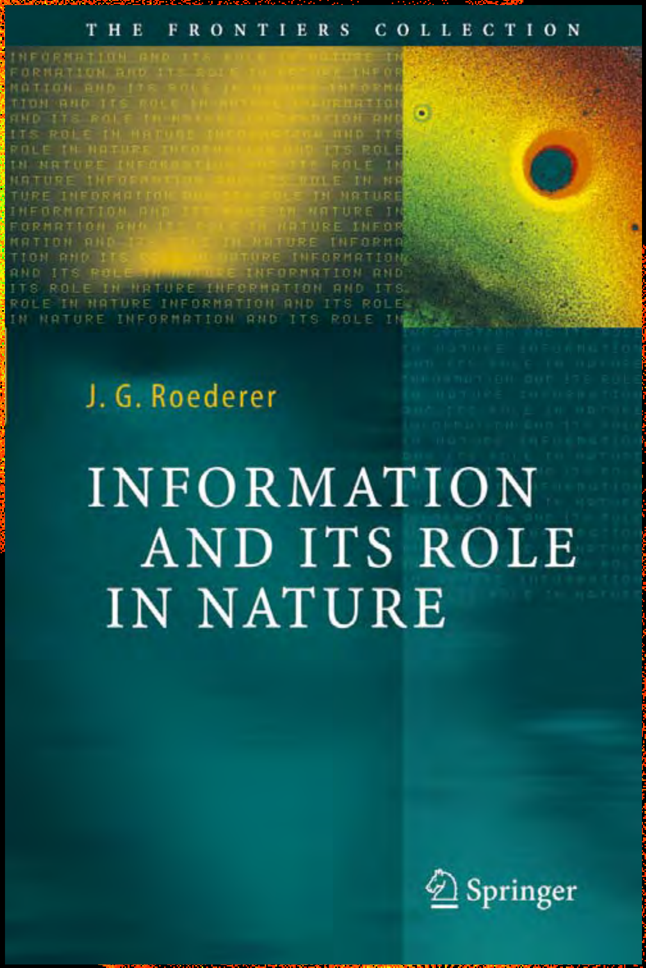
The TIFF on THREE SCALES



Pictures are more democratic
but they come from formulae

Roots in the most stable colouring

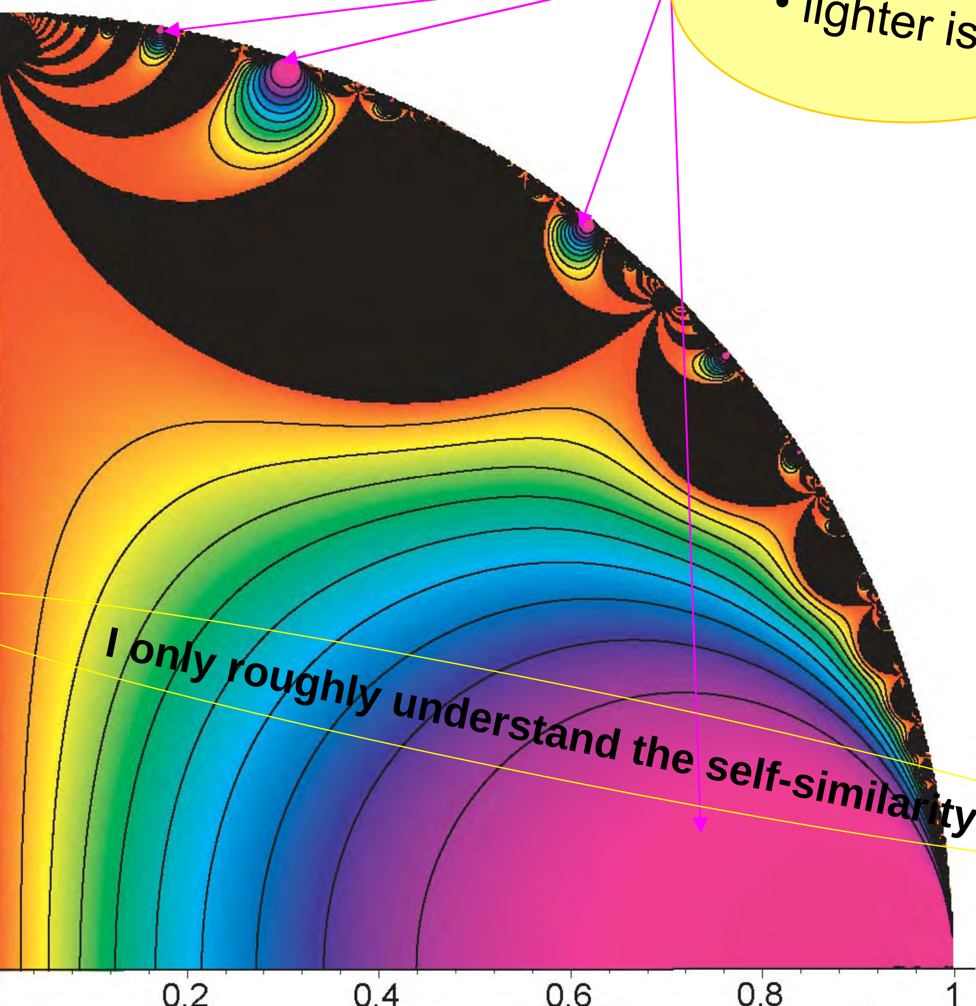


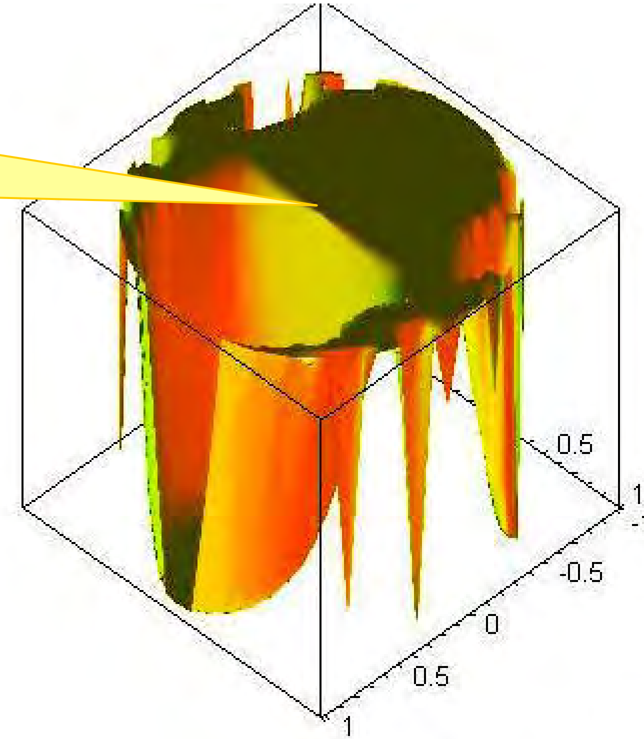


FRACTAL of a Modular Inequality

$$\mathcal{R} = \frac{\left|\sum_{n\in\mathbf{Z}}(-1)^n q^{n^2}\right|}{\left|\sum_{n\in\mathbf{Z}} q^{n^2}\right|}$$

plots $\mathcal{R}$ in disk
- black exceeds 1
- lighter is lower



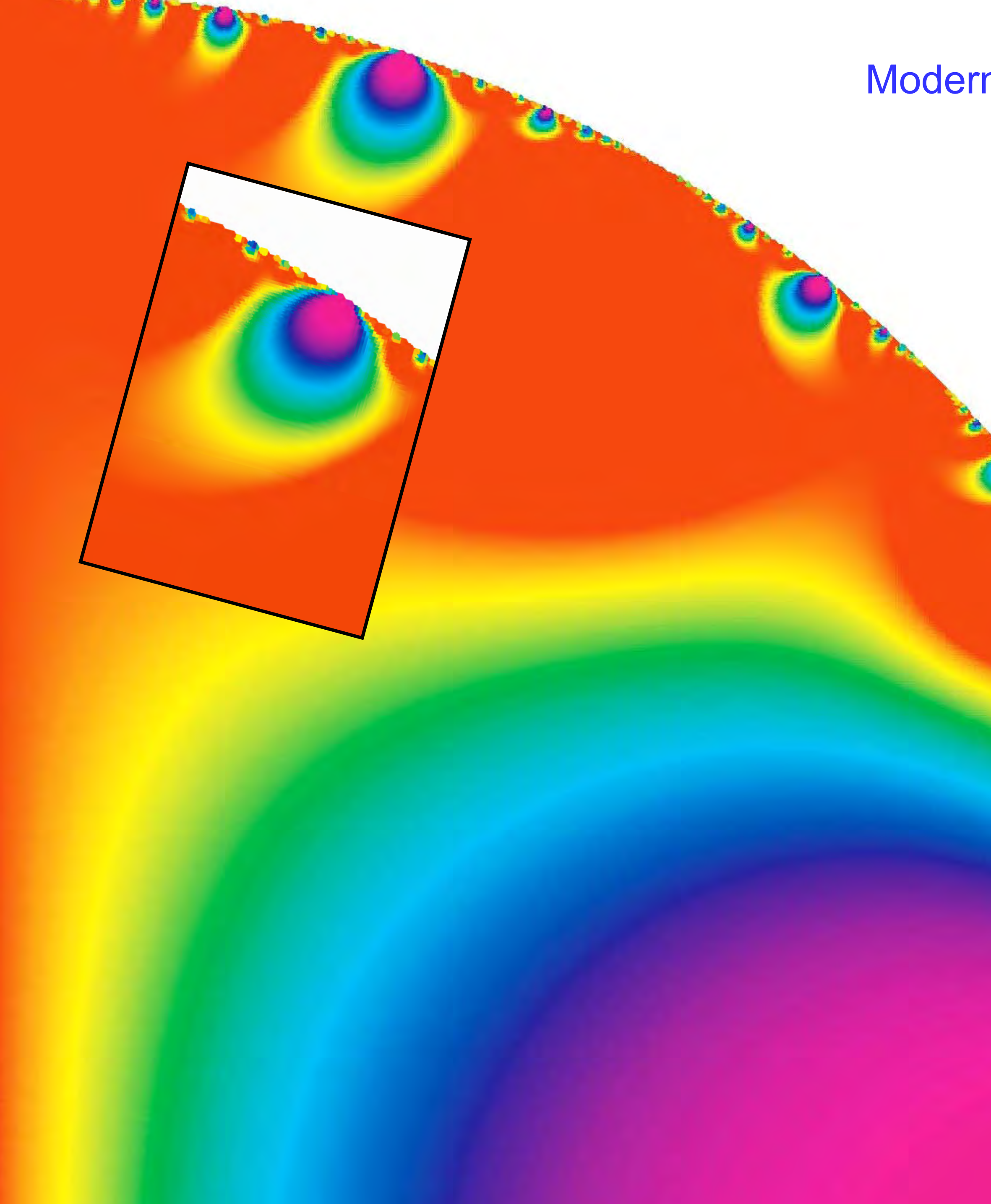


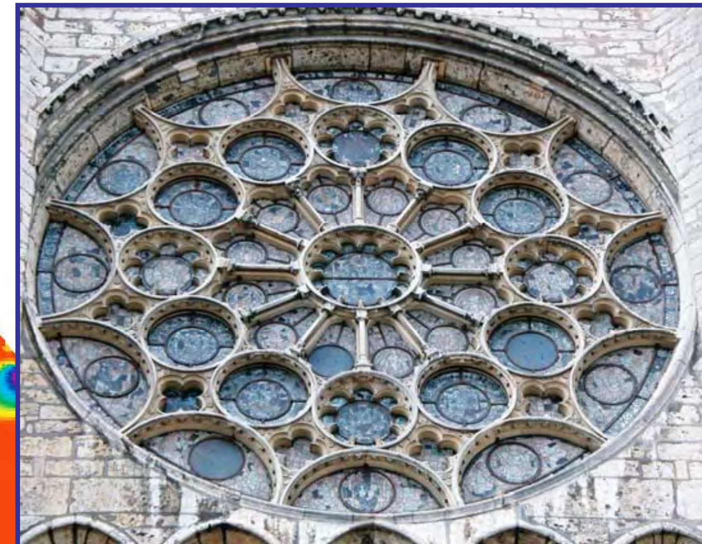
- ✓ related to Ramanujan's continued fraction
- ✓ took several hours to print
- ✓ Crandall/Apple has parallel print mode

Mathematics and the aesthetic

Modern approaches to an ancient affinity

(CMS-Springer, 2005)



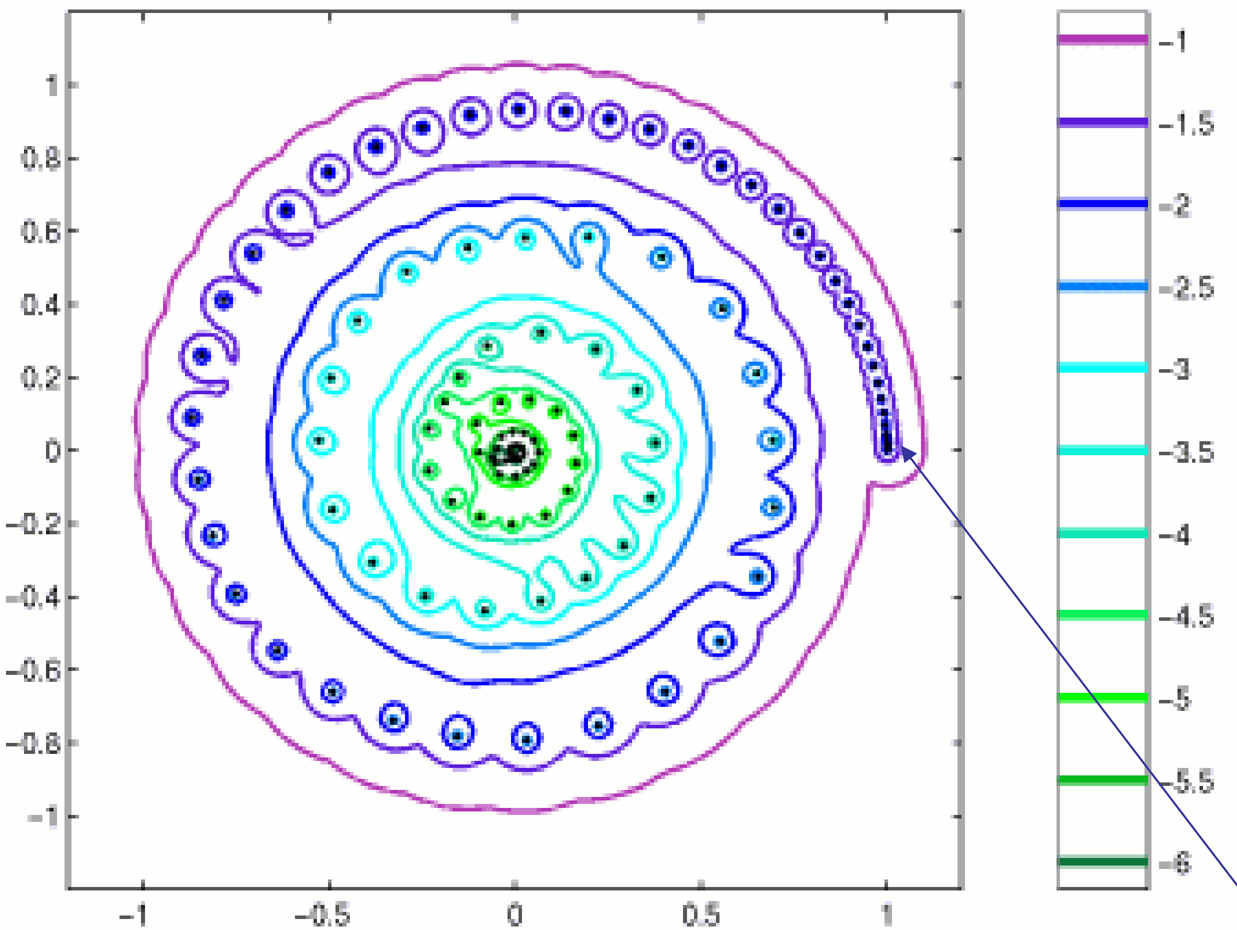


Why should I refuse a good dinner simply because I don't understand the digestive processes involved?

Oliver Heaviside (1850 - 1925)

✓ **when criticized for his daring use of operators before they could be justified formally**

An Early Use of Pseudospectra (Landau, 1977)



ε =10^t

An infinite dimensional integral equation in laser theory

✓discretized to a matrix of dimension **600**

✓projected onto a well chosen invariant subspace of dimension **109**

Generic Code Optimization

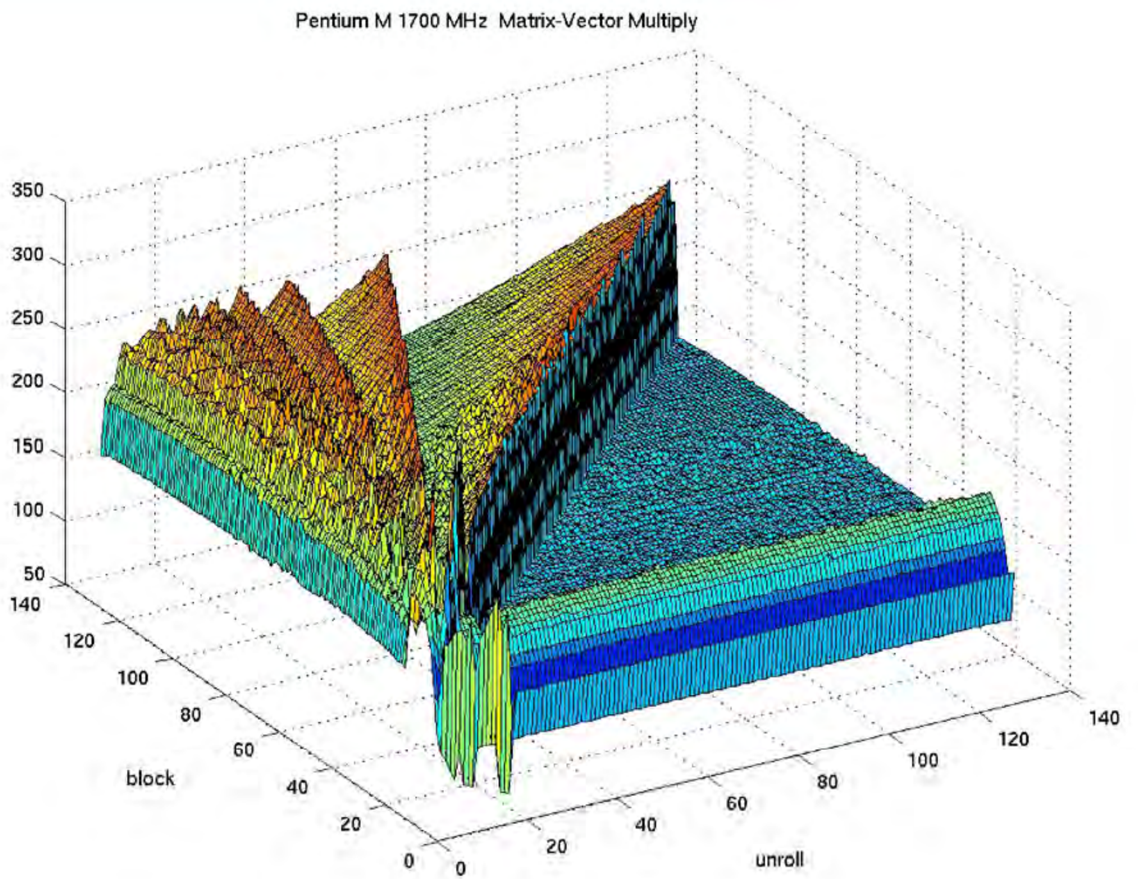
Experimentation with DGEMV (matrix-vector multiply):

128x128=16,384 cases.

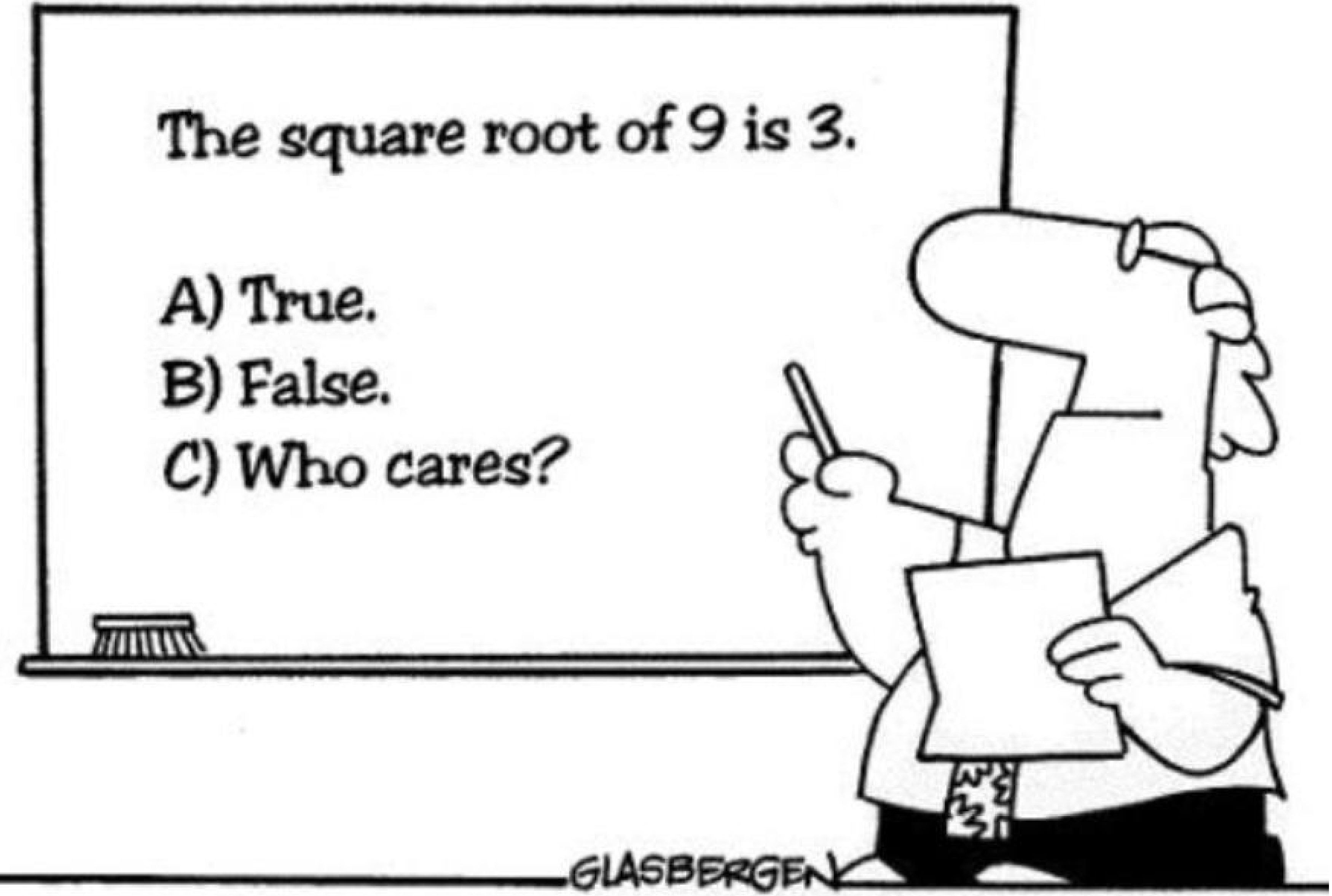
Experiment took 30+ hours to run.

Best performance = 338 Mflop/s with blocking=11 unrolling=11

Original performance = 232 Mflop/s

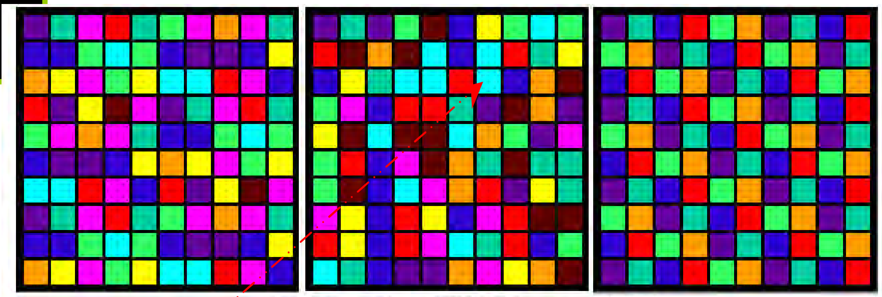


Visual Representation of Automatic Code Parallelization



Many students actually look forward to Mr. Atwadder's math tests.

IVa. Inverse & Color Calculators

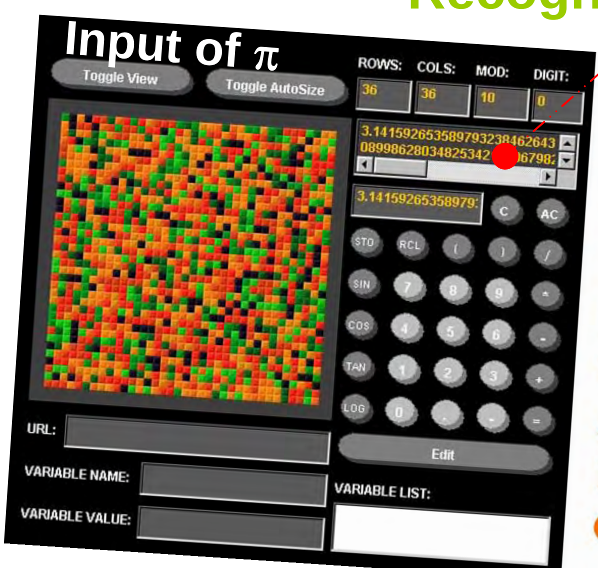


Archimedes: $223/71 < \pi < 22/7$

Inverse Symbolic Computation

- "**Inferring symbolic structure from numerical data**"
- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs ***micro-parallelism***
- It faces the "curse of exponentiality"

➢ Implemented as **identify** in Maple and **Recognize** in Mathematica

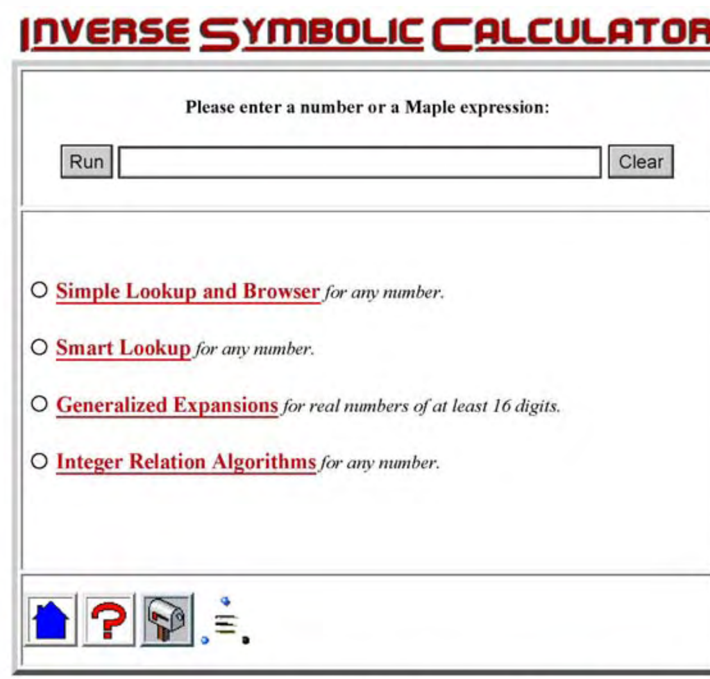


COLORCALC

identify(sqrt(2.)+sqrt(3.))

3.146264370

$$\sqrt{2}+\sqrt{3}$$



Expressions that are **not** numeric like ln(Pi*sqrt(2)) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.

ENTERING

evalf(Sum(k^k/k!/exp(k)-1/sqrt(2*Pi*k),k=1..infinity),16)

'Simple Lookup' fails;
'Smart Look up' gives:

INVERSE SYMBOLIC CALCULATOR

The ISC is the **Inverse Symbolic Calculator**, a set of programs and specialized tables of mathematical constants dedicated to the identification of real numbers. It also serves as a way to produce identities with functions and real numbers. It is one of the main ongoing projects at the Centre for Experimental and Constructive Mathematics (CECM).

Results of the search:

Maple output:

.08406950872765600

.8406950872765600e-1

Value to be looked up: .8406950872765600e-1 = K

Performing a smart lookup on .8406950872765600e-1:

Function	Result	Precision	Matches
K-2/3	.58259715793901066666666666	16	1

INVERSE SYMBOLIC CALCULATOR

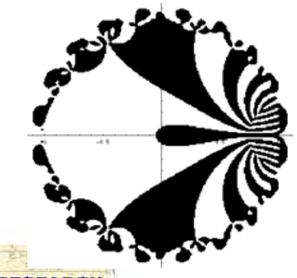
579390106 was probably generated by one
s or found in one of the given tables.
Answers are given from shortest to longest description

Mixed constants with 5 operations
5825971579390106 = Zeta(1/2)/sr(2)/sr(Pi)

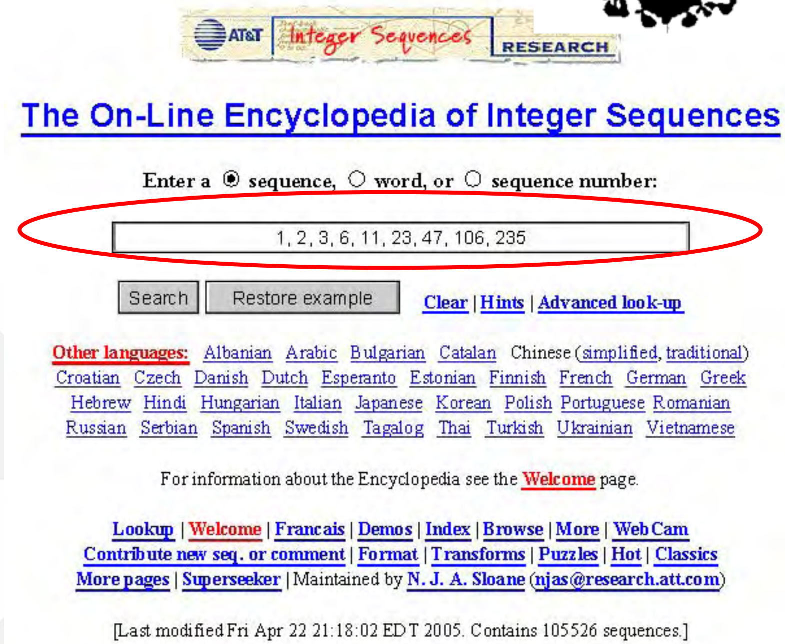
Browse around .5825971579390106.

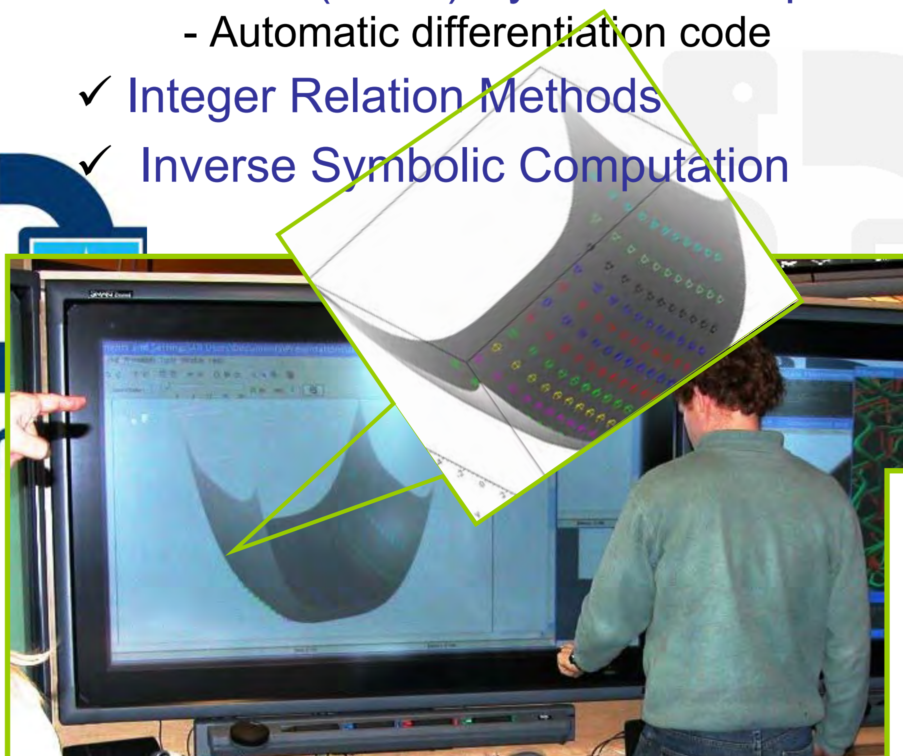
"Just a darn minute! — Yesterday you said that X equals two!"

IVb. Numeric and Symbolic Computation



- ❑ <u>Central to my work</u> - with Dave Bailey - meshed with visualization, randomized checks, many web interfaces and
  - ✓ Massive (serial) Symbolic Computation
    - Automatic differentiation code
  - ✓ Integer Relation Methods
  - ✓ Inverse Symbolic Computation





Parallel derivative free optimization in ***Maple***

<u>Other useful tools</u> : Parallel Maple

- Sloane's online sequence database
- Salvy and Zimmerman's generating function package '*gfun*'
- Automatic identity proving: Wilf-Zeilberger method for hypergeometric functions

Peter Borwein
in front of
Helaman Ferguson's
work

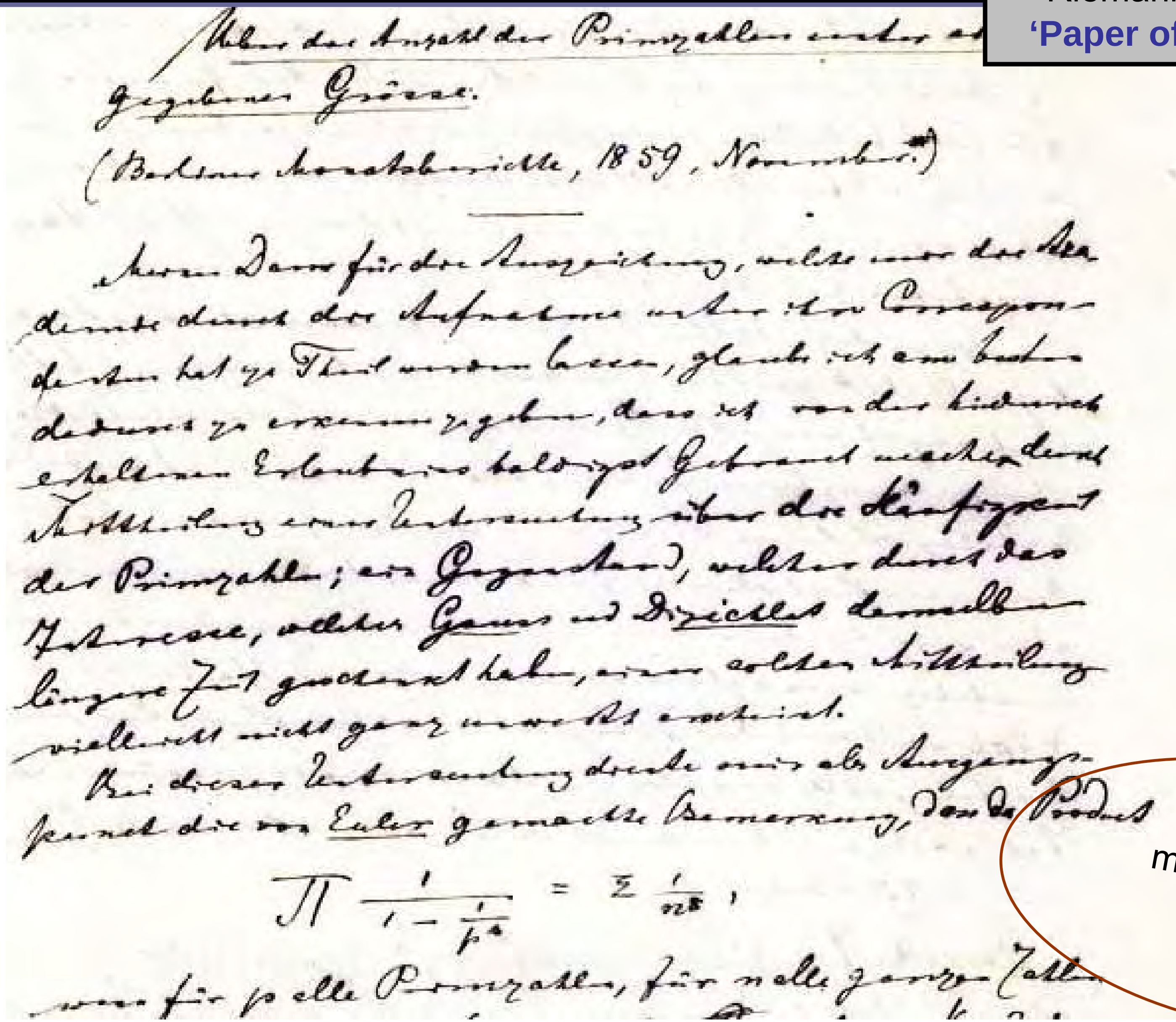
CMS Meeting
December 2003
SFU Harbour Centre

Ferguson uses high tech tools and micro engineering at NIST to build monumental math sculptures

Uber die Anzahl der Primzahlen unter einer Gegebenen Grosse

On the number of primes less than a given quantity

Riemann's six page 1859 **'Paper of the Millennium'?**



Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse.

(Berliner Monatsberichte, 1859, November.)

Meinen Dank für die Auszeichnung, welche mir die Akademie durch die Aufnahme unter ihre Correspondenten hat zu Theil werden lassen, glaube ich am besten dadurch zu erkennen zu geben, dass ich von der hierdurch erhaltenen Erlaubniss baldigst Gebrauch mache durch Mittheilung einer Untersuchung über die Häufigkeit der Primzahlen; ein Gegenstand, welcher durch das Interesse, welches Gauss und Dirichlet demselben längere Zeit geschenkt haben, einer solchen Mittheilung vielleicht nicht ganz unwerth erscheint.

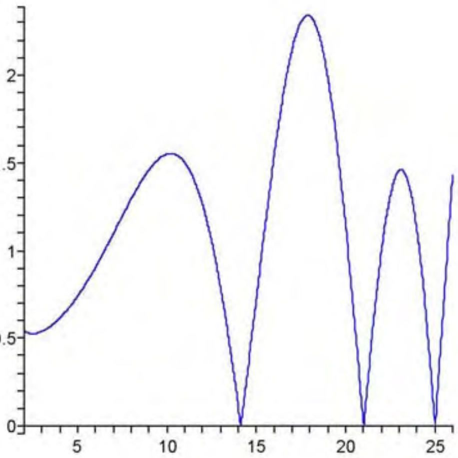
Bei dieser Untersuchung diente mir als Ausgangspunkt die von Euler gemachte Bemerkung, dass das Product

$$\prod \frac{1}{1-\frac{1}{p^s}} = \sum \frac{1}{n^s},$$

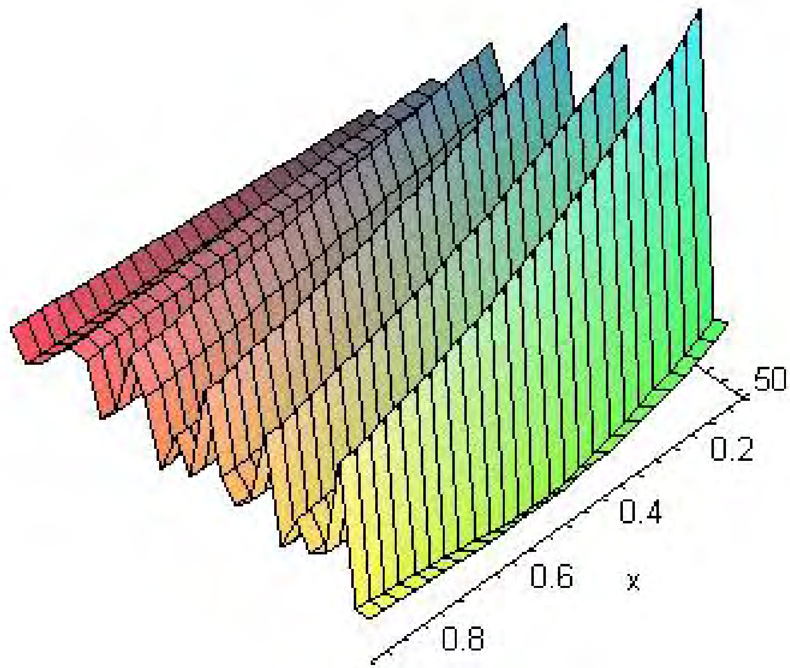
wenn für p alle Primzahlen, für n alle ganzen Zahlen

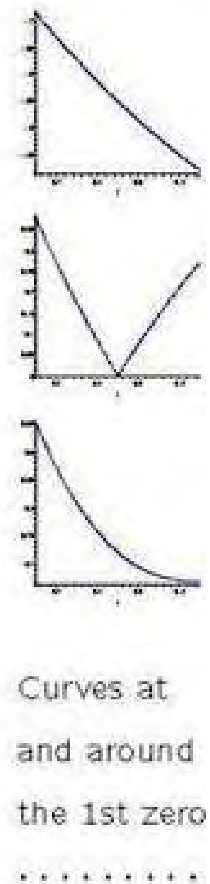
RH is so important because it yields precise results on distribution and behaviour of primes

Euler's product makes the key link between **primes and ζ**



The Modulus of Zeta and the **Riemann Hypothesis** (A Millennium Problem)





The imaginary parts of first 4 zeroes are:

14.134725142
21.022039639

25.010857580
30.424876126

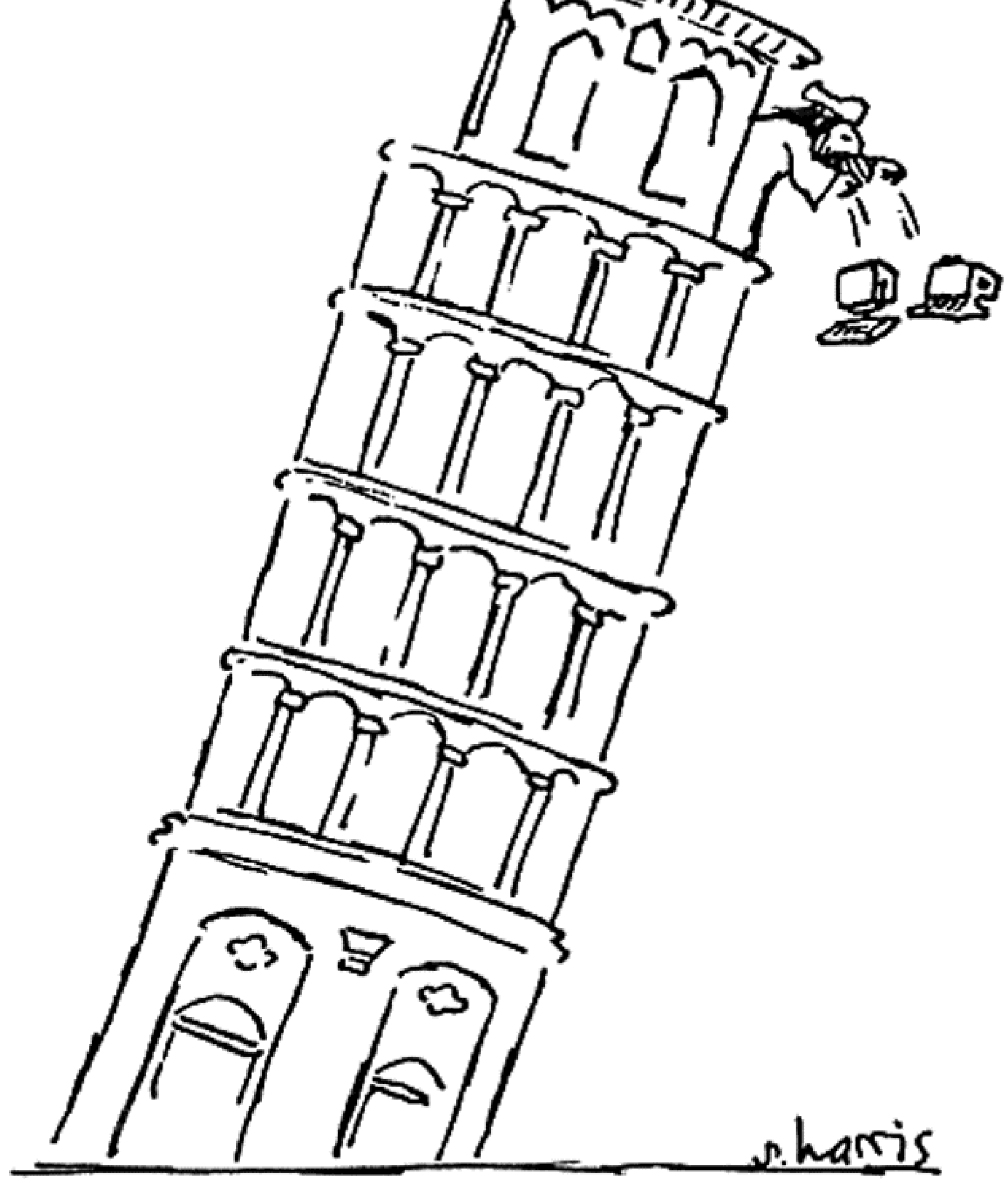
The first 1.5 billion are on the *critical line*

Yet at 10^{22} the ***"Law of small numbers"*** still rules (Odlyzko)

'All non-real zeros have real part one-half'

(The Riemann Hypothesis)

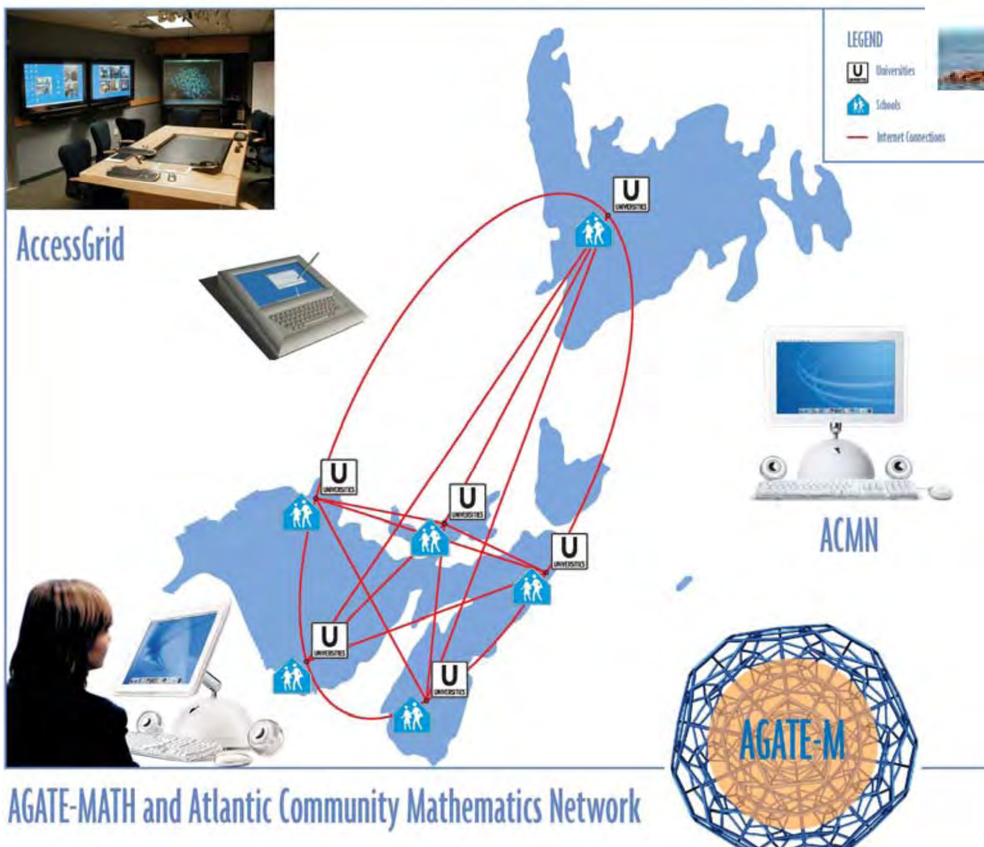
Note the **monotonicity** of **|ζ(¢+iy)|** is **equivalent to RH** (discovered in a Calgary class in 2002 by Zvengrowski and Saidak)

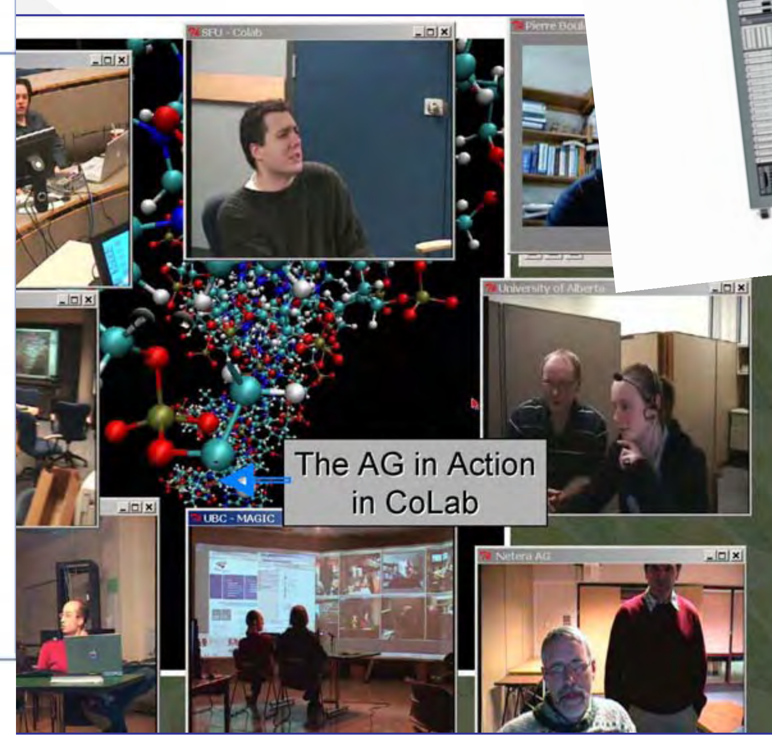


IF THERE WERE COMPUTERS IN GALILEO'S TIME

V. Access Grid, AGATE and Apple

First 25 teachers identified





Atlantic Gateway to Mathematics

AGATE-MATH was recently established for the purpose of improving, encouraging, and supporting the teaching of mathematical sciences, in Atlantic Canada and elsewhere.

Vision Statement

The discipline of Mathematics is beautiful and important in its own right. At the same time mathematics and mathematical competency are critical to most other scientific disciplines and are pervasive in modern society. Cell phones, Google, e-banking, internet security, "Finding Nemo," all use enormously sophisticated mathematics, as do countless more obvious examples from medical imaging to mutual funds.

Mathematics is a fundamental component of the language of science. Consequently, mastery of basic mathematics is critical for sustaining interest not only in the pursuit of science but also in understanding the sciences (physical, biological, artificial, social and human) that affect our lives. Successful scientists and engineers typically report a serious early engagement with mathematics as one of their formative experiences. Base competency and interest in mathematics and science are often achieved or lost before the end of high school and likely by the end of elementary grades.

Goals of AGATE-M

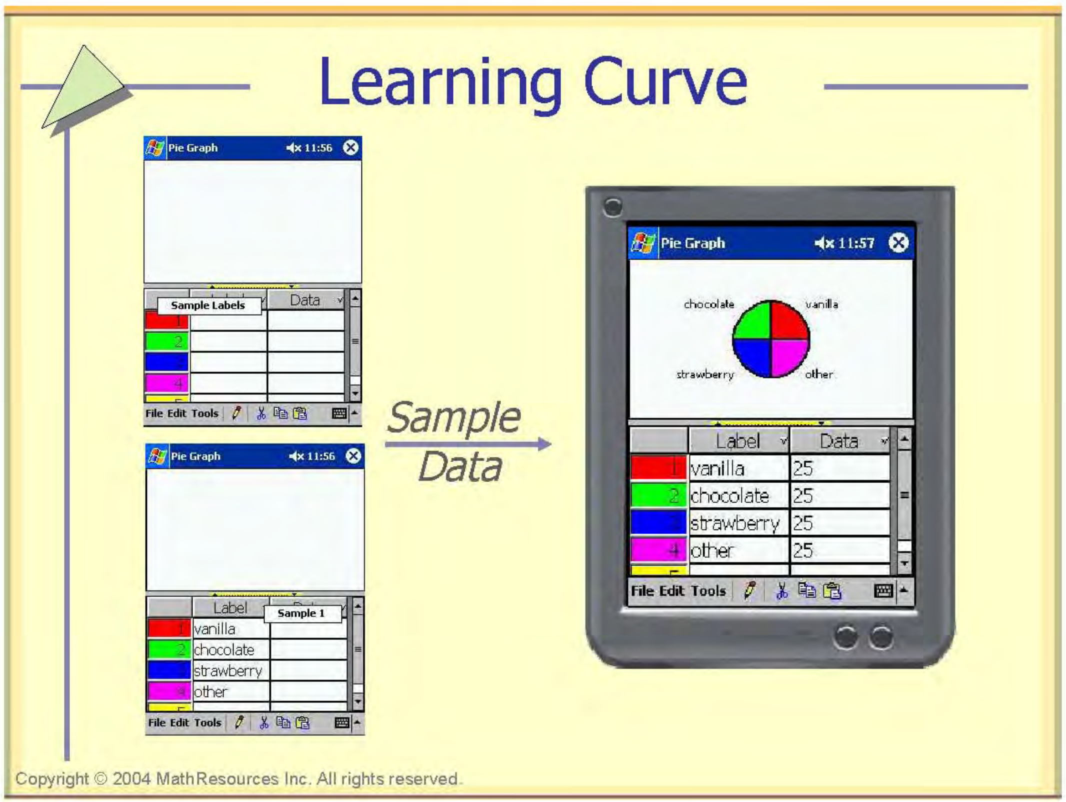
- To create a network linking everyone with an interest in math education.
- To enable easy communication between teachers and researchers.
- To strengthen the sense of community amongst those who share the goal of improving math education.
- To provide a forum for the discussion of current issues.
- To offer enrichment resources through web based resources.
- To facilitate the dissemination of knowledge and experience.
- To stimulate enthusiasm and creative thinking in our community.


THEORIES
PROOFS

VI. University – Industry links

MITACS – MRI

putting high end science on a hand held




D4 Wednesday, December 15, 2004 BUSINESS

Try your hand at new math

Firm develops software to help guide kids through maze of numbers

By GREG MacVICAR

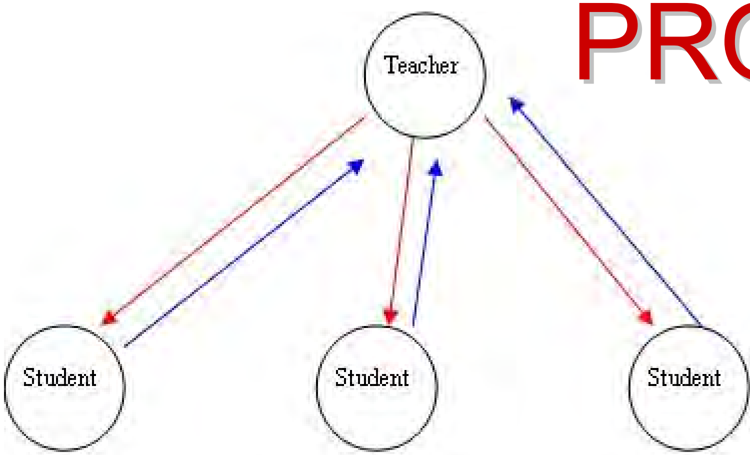
Ron Fitzgerald says math is a language — and most students are illiterate.

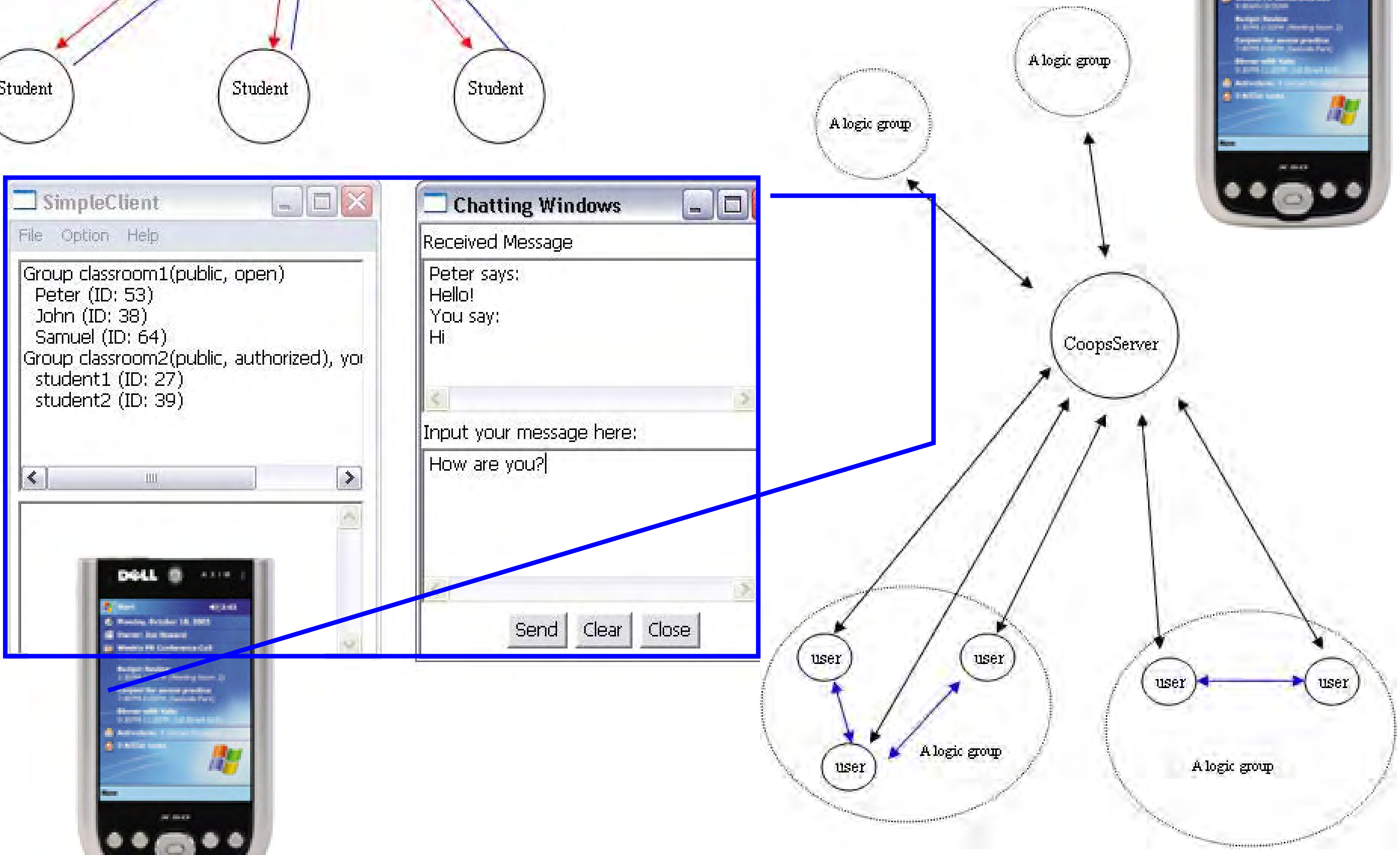
The president of Halifax software company MathResources Inc. wants to change that. That's why Mr. Fitzgerald and his wife quit their jobs as book editors in Toronto in 1994.

Ten years later, he says his company …

Ronald Fitzgerald, president of MathResources Inc., holds a hand-held computer capable of … conventional computers and running the company's mathematics programs.

PROTOTYPE for handheld collaboration





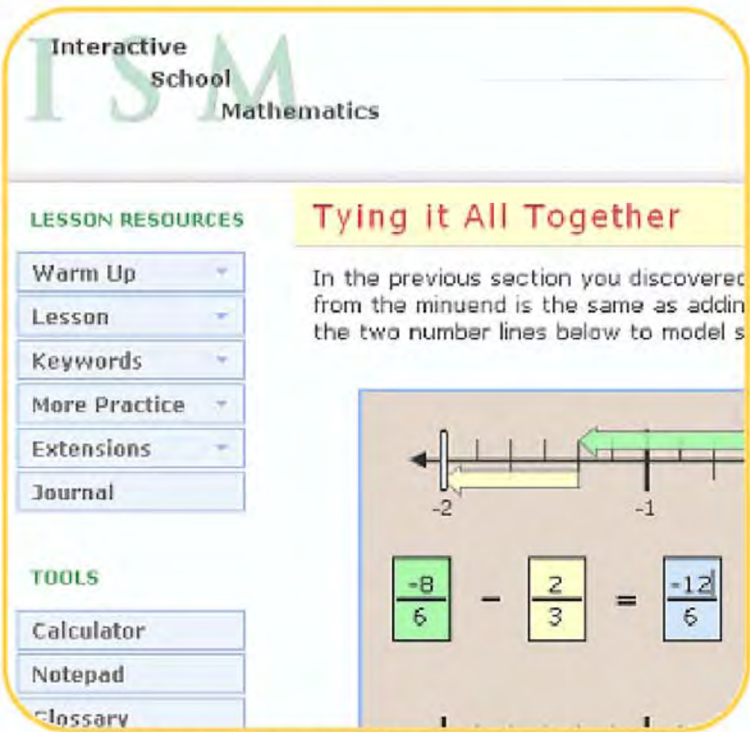
MathResources Inc.

Interactive School Mathematics

MRI's Newest Product

Attention: Schools and Districts Pursuing One-to-One Computing Initiatives

Improve Student Achievement in Mathematics,
Save Teachers Time, and Reduce the Overall Costs of Instruction



Above: A portion of an ISM lesson.

Interactive School Mathematics is a curriculum that fully integrates technology into math programs. By making computers central to teaching and learning, ISM eliminates the need for textbooks and calculators.

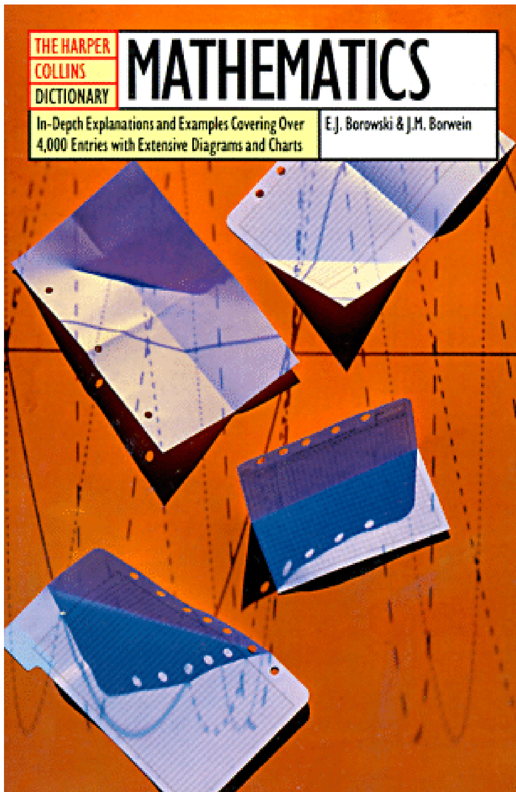
Follow these links to:

- Discover ISM's significant benefits
- Experience complete sample lessons
- Meet ISM's distinguished authors
- Review ISM's curriculum

Pilot ISM in Your District

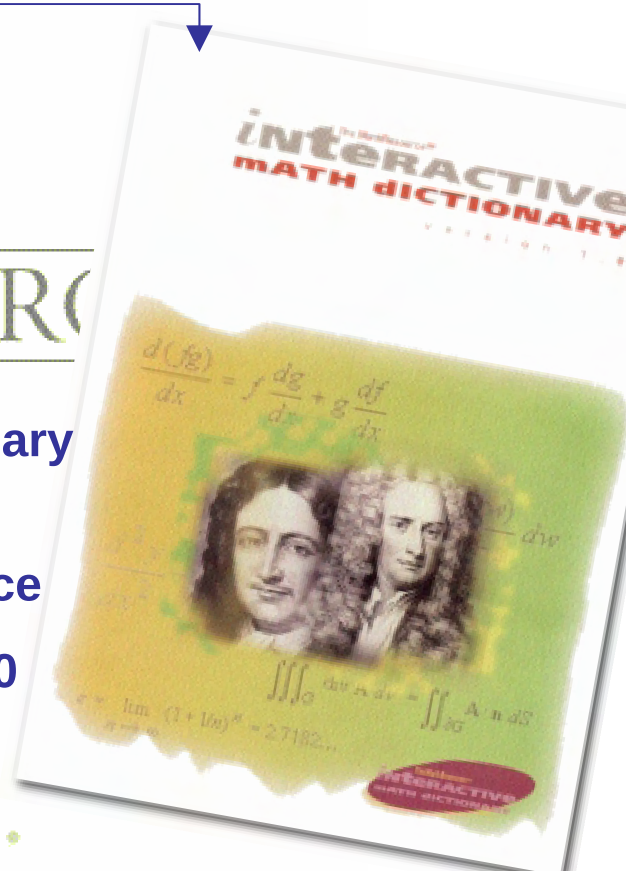
- Find out how

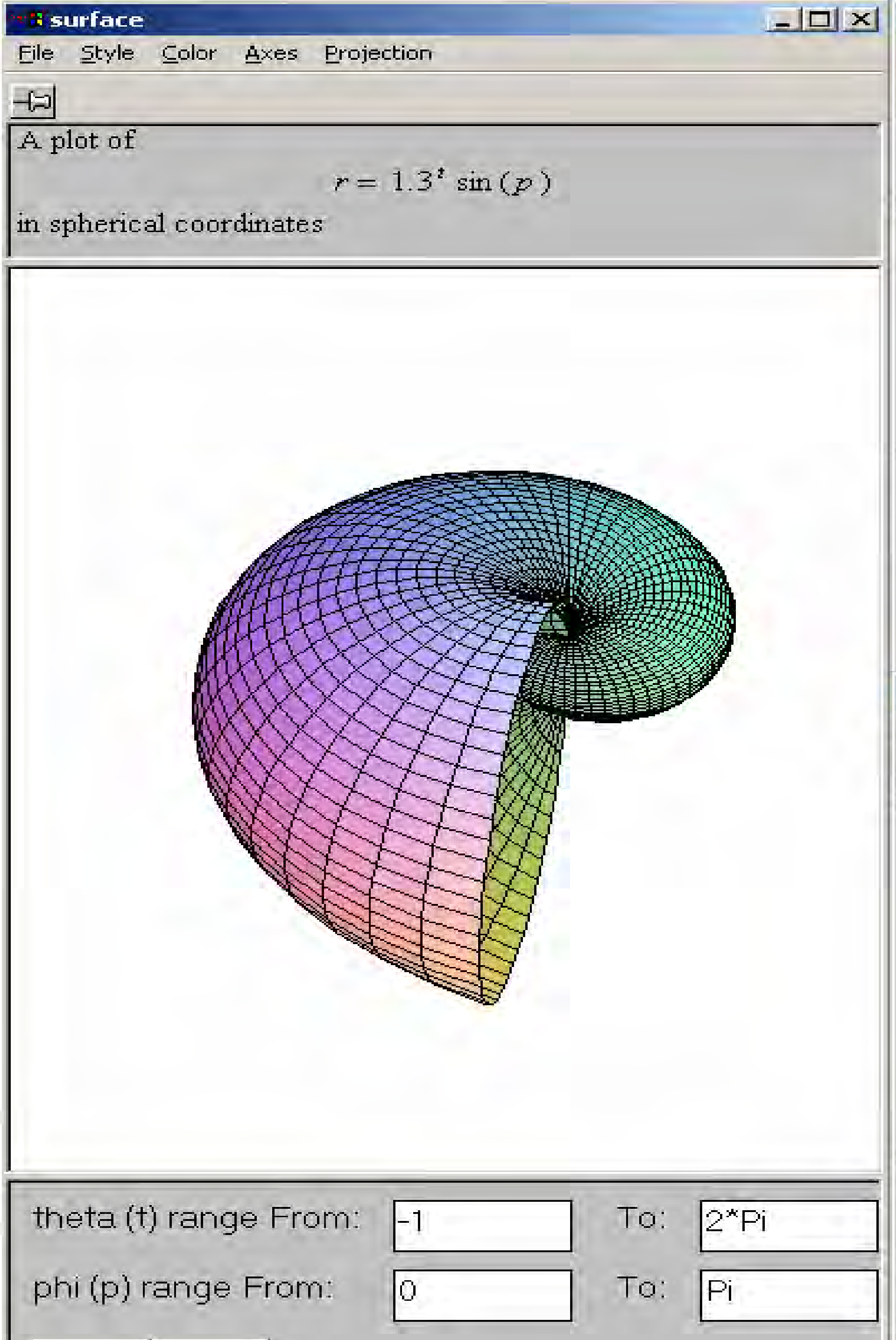
MRI's First Product in Mid-nineties

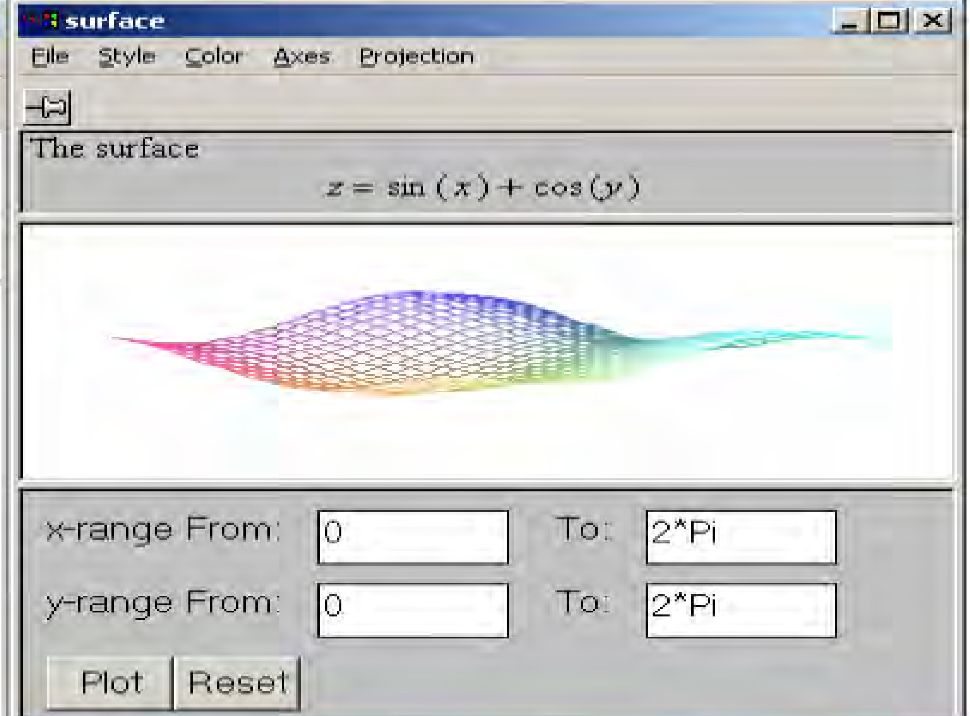


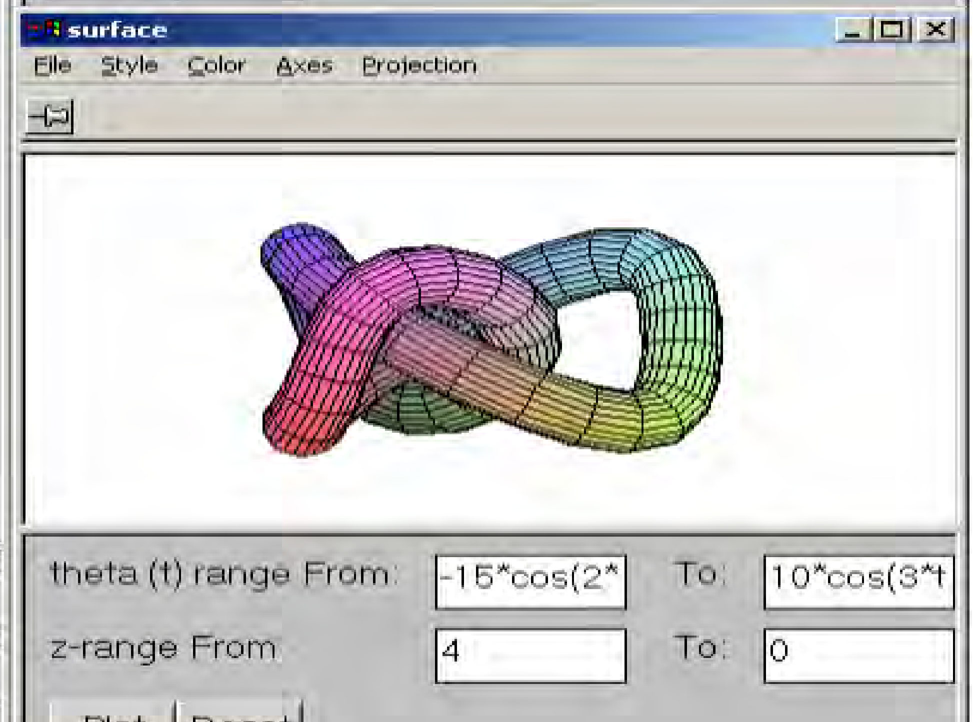
Maplesoft

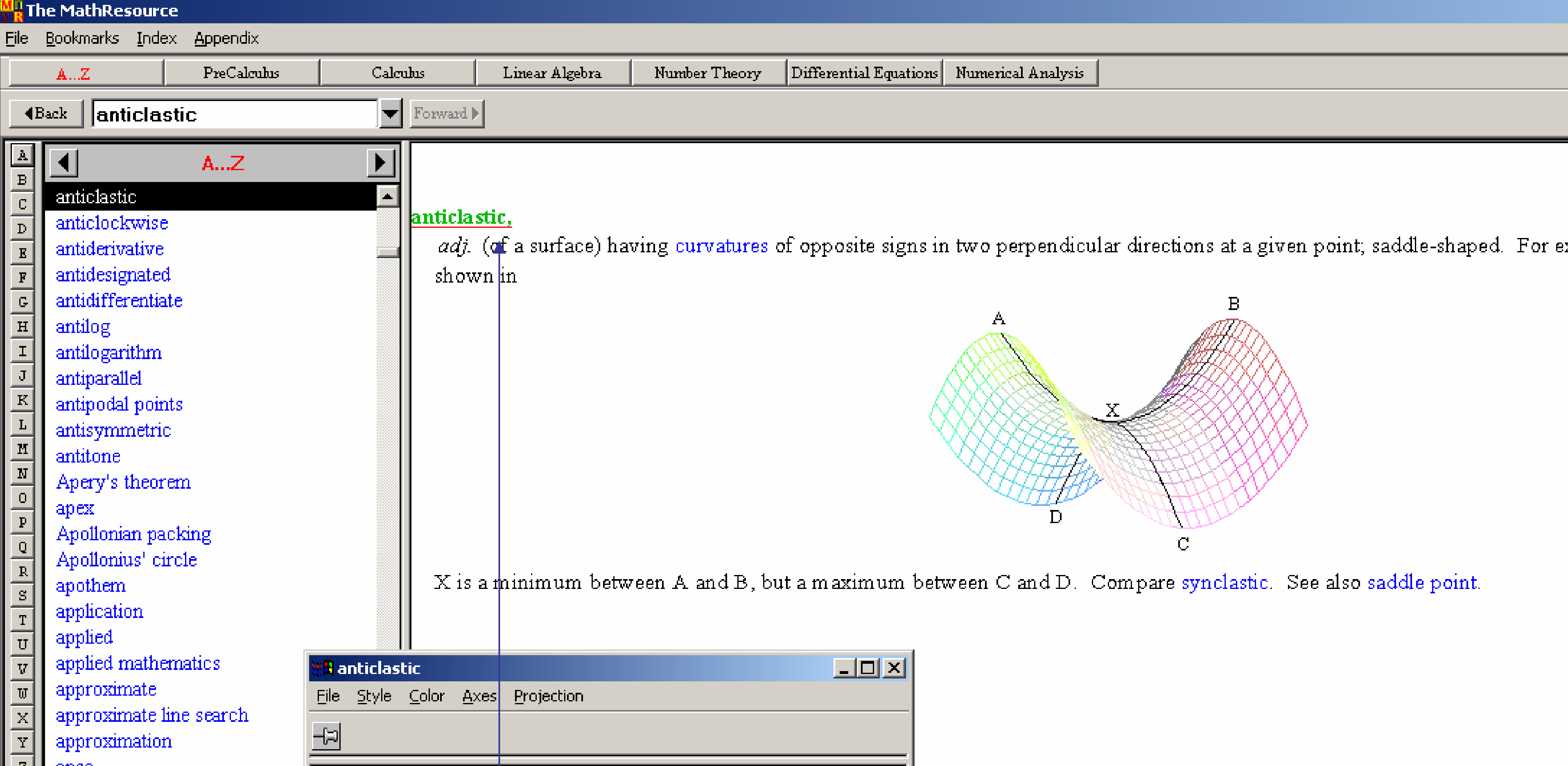
- ► Built on **Harper Collins dictionary** - an IP adventure!
- ► **Maple** inside the **MathResource**
- ► **Data base** now in **Maple 9.5/10**
- ► **CONVERGENCE?**

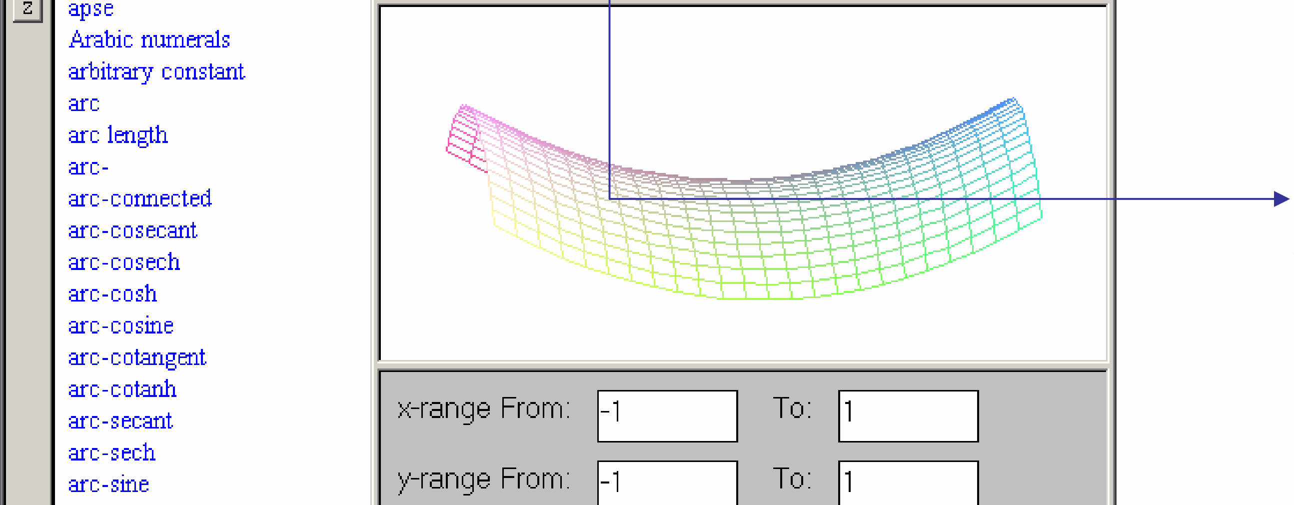



surface
File Style Color Axes Projection
A plot of
r = 1.3^t sin(p)
in spherical coordinates
theta (t) range From: -1 To: 2*Pi
phi (p) range From: 0 To: Pi


surface
File Style Color Axes Projection
The surface
z = sin(x) + cos(y)
x-range From: 0 To: 2*Pi
y-range From: 0 To: 2*Pi
Plot Reset


surface
File Style Color Axes Projection
theta (t) range From: -15*cos(2* To: 10*cos(3*t
z-range From 4 To: 0





- Any blue is a hyperlink
- Any green opens a reusable Maple window with initial parameters set
- Allows exploration with no learning curve

Building on products such as:

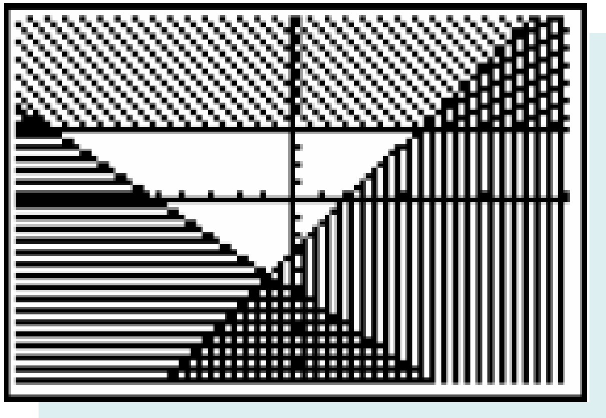
MRI Graphing Calculator & Robert Morris College

Ed Clark, an instructor at Robert Morris College, has been using the MRI Graphing Calculator with his students. Ed says:

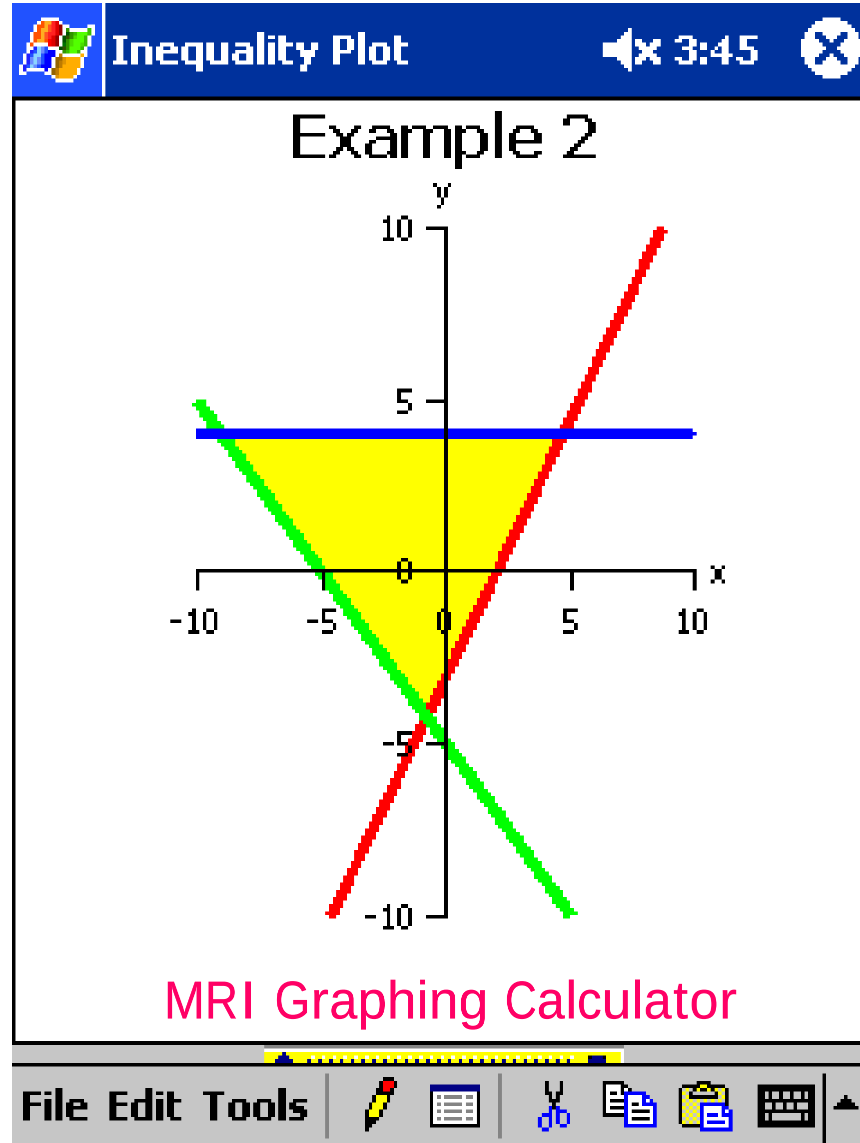
- “The learning curve for the MRI Graphing Calculator is very very short.”
- “Just the fact that a handheld computer displays color is huge.”

MathResources Inc.

Graphing in Color

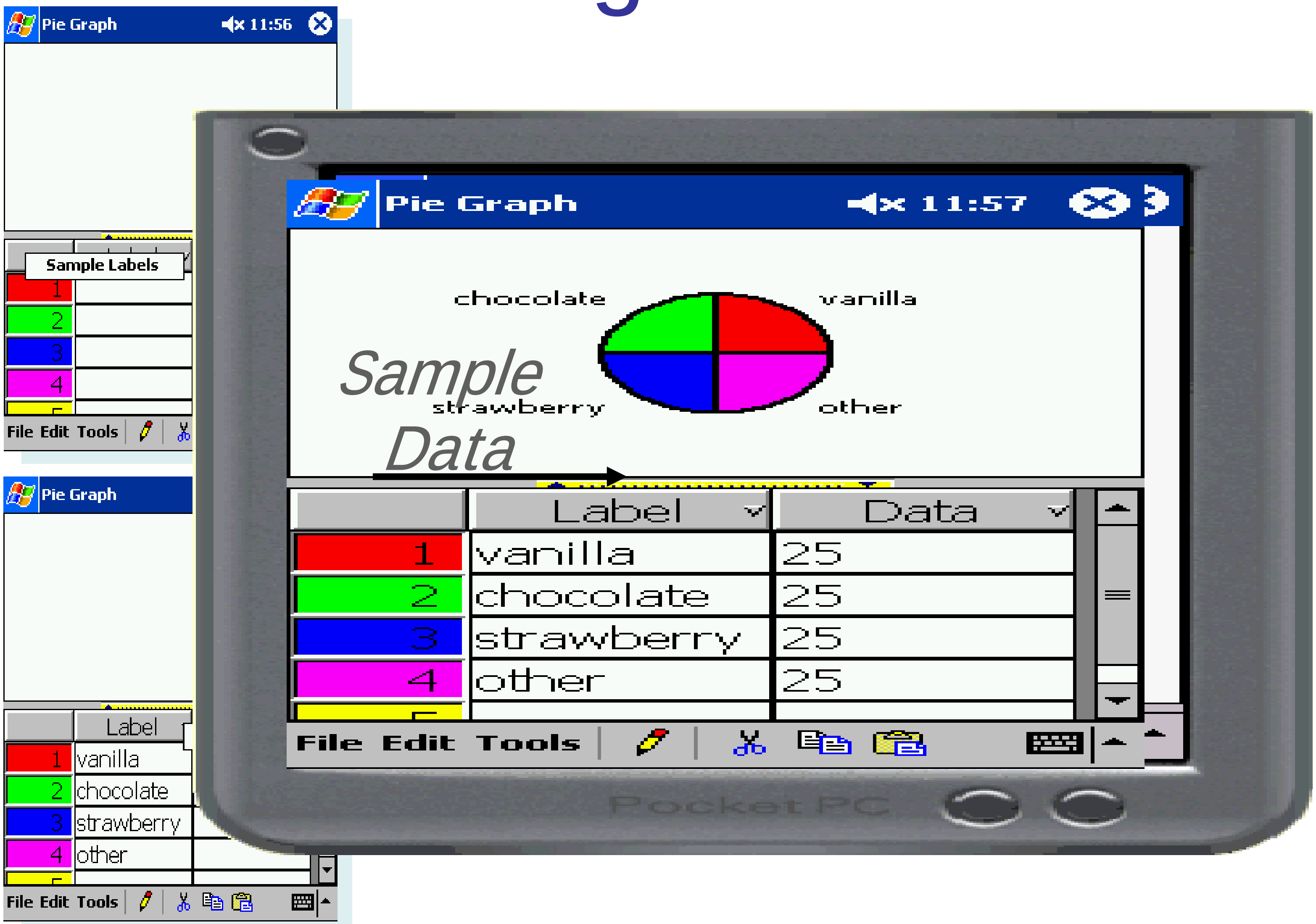


Traditional
Graphing Calculator



MRI Graphing Calculator

Learning Curve



A selection of appropriate virtual **manipulables**

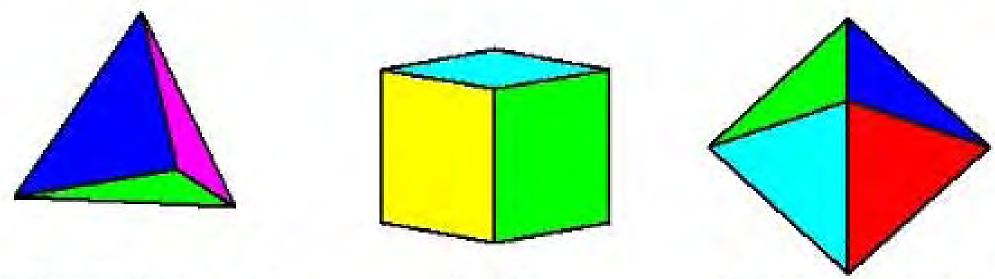
Index

Platonic solids

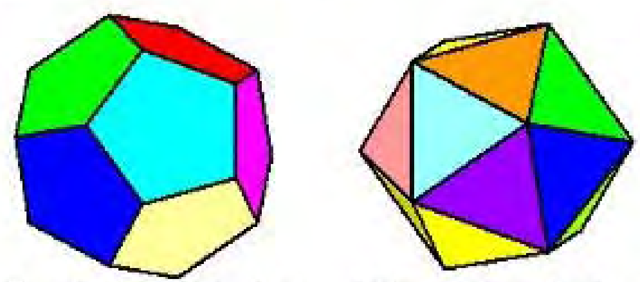
Also called regular polyhedra.

The five special polyhedra where all of the faces of each polyhedron are congruent regular polygons and the same number of polygons meet at each vertex. The ancient Greeks proved that there are only five platonic solids. They are: cube, tetrahedron, octahedron, dodecahedron, and icosahedron.

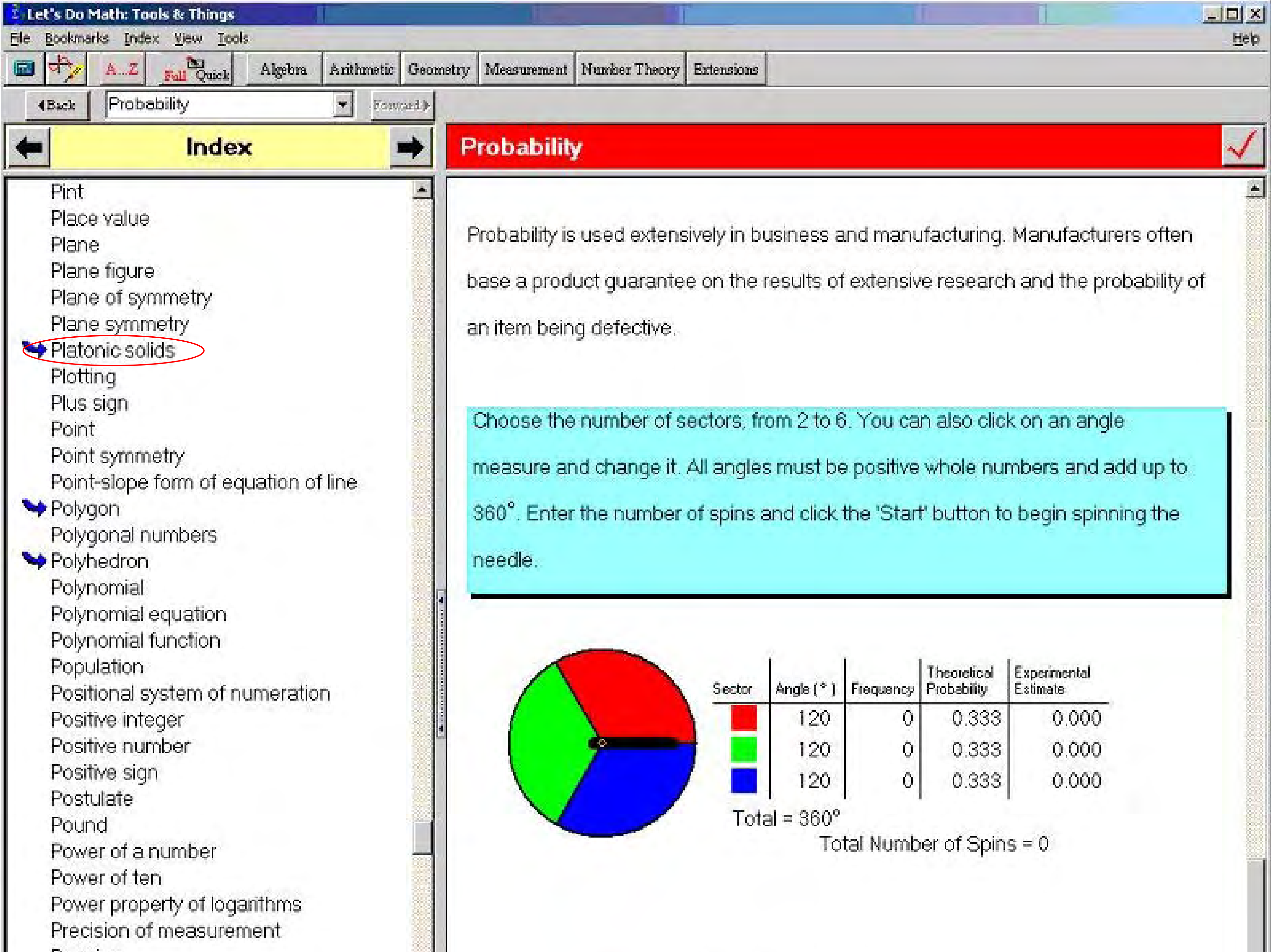
Click on one of the polyhedra below and drag the mouse to rotate it. By right clicking on one of the polyhedra you can change to a wire frame view.



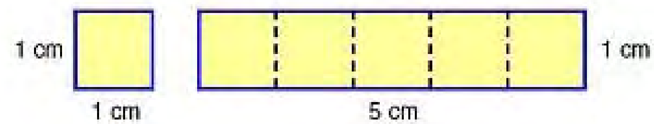
A Regular Tetrahedron   A Cube   A Regular Octahedron



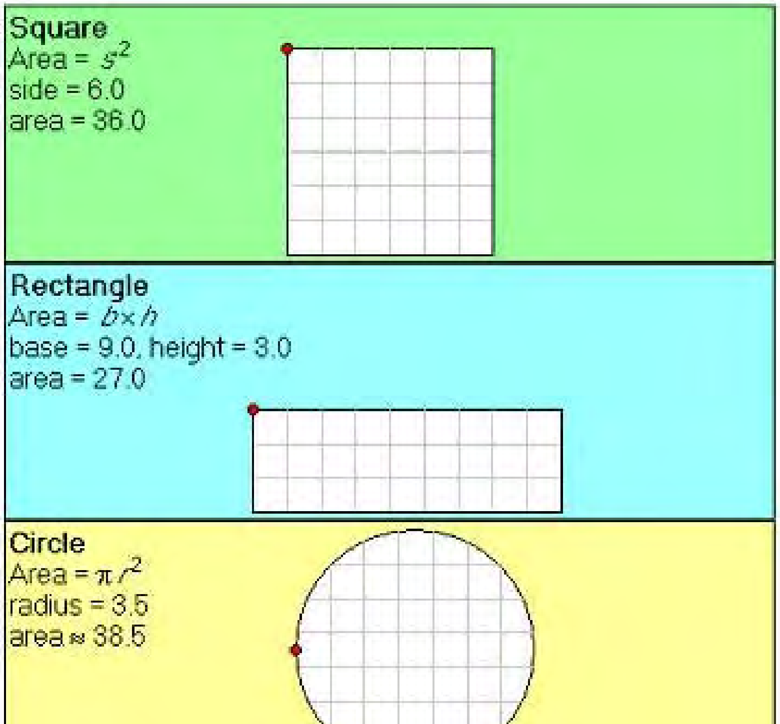
A Regular Dodecahedron   A Regular Icosahedron


Let's Do Math: Tools & Things
File Bookmarks Index View Tools Help
A...Z Full Quick Algebra Arithmetic Geometry Measurement Number Theory Extensions
Back Probability Forward
Index
Probability
Pint
Place value
Plane
Plane figure
Plane of symmetry
Plane symmetry
Platonic solids
Plotting
Plus sign
Point
Point symmetry
Point-slope form of equation of line
Polygon
Polygonal numbers
Polyhedron
Polynomial
Polynomial equation
Polynomial function
Population
Positional system of numeration
Positive integer
Positive number
Positive sign
Postulate
Pound
Power of a number
Power of ten
Power property of logarithms
Precision of measurement
Probability is used extensively in business and manufacturing. Manufacturers often base a product guarantee on the results of extensive research and the probability of an item being defective.
Choose the number of sectors, from 2 to 6. You can also click on an angle measure and change it. All angles must be positive whole numbers and add up to 360°. Enter the number of spins and click the 'Start' button to begin spinning the needle.
Sector
Angle (°)
Frequency
Theoretical Probability
Experimental Estimate
120 0 0.333 0.000
120 0 0.333 0.000
120 0 0.333 0.000
Total = 360°
Total Number of Spins = 0

The amount of space within a two-dimensional figure. It is usually measured in square units. The square below has an area of one square centimetre, 1 cm^2. It takes exactly 5 of these to cover the rectangle, which tells you that the area of the rectangle is 5 cm^2.



Drag the points on the figures below to see how their area changes.



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J.M. Borwein, ``The Experimental Mathematician: The Pleasure of Discovery and the Role of Proof," *International Journal of Computers for Mathematical Learning*, **10** (2005), 75--108.

D.H. Bailey and J.M Borwein, "Experimental Mathematics: Examples, Methods and Implications," *Notices Amer. Math. Soc.*, **52** No. 5 (2005), 502-514.


Enigma

“The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.”

• J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.