Mathematics 2600
Second Midterm Test, September 25, 2008

There are **ten multiple choice** questions worth 1 point and **four Maple tasks** worth 2.5 points: for a **total of 20 points.**

    NO stands for "none of the previous".

Part A: Multiple Choice: answer on this paper

1. The instruction **> matrix(2,3,[seq(k^3,k=1..9)]);** will produce a matrix with

A. 3 rows, 2 columns   B. 3 rows, 3 columns   C. 2 rows, 3 columns   D. NO

2. Determine the determinant of the matrix

$$M := \begin{bmatrix} 2 & 5 & 10 & 17 \\ 26 & 37 & 50 & 65 \\ 82 & 101 & 122 & 145 \\ 170 & 197 & 226 & 257 \end{bmatrix}$$

    A. 0   B. 1   C. 16!   D. NO

3. The inequality $x\sin(1/x^2) \le x$ holds for

A. All real x.   B. $x > 0$ C. $x \ge 0$ D. NO

4. The integral $\dfrac{1}{2}\displaystyle\int_{-\infty}^{\infty} \dfrac{\sin(x)^2}{x^2}dx$ is

A. π B. $\pi/2$ C. $1.570796327...$ D. NO

5. The sum $\displaystyle\sum_{k=0}^{11} k^{10}$ is

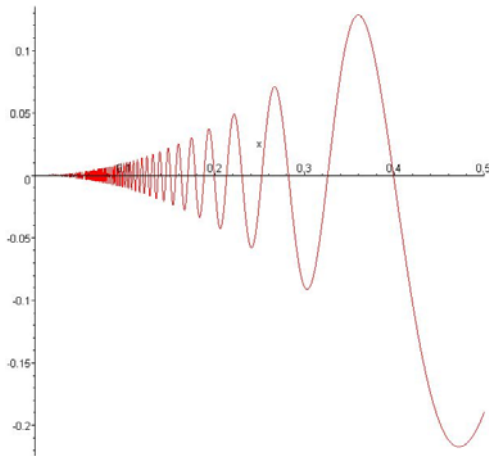A. 14914341925.0   B. 142364319625   C. 14914341925   D. NO

6. The largest prime less than a billion is

A. 9999999999989   B. 99999999989   C. 99999999987   D. NO

7. The picture below is the graph for $0 < x < 1/2$ of

A. $x \sin(1/x)$ B. $x^2 \sin(1/x^2)$ C. $x^2 \sin(1/x)^2$ D. NO

8. The value of $\lim\limits_{x \to \infty} \left(1 + \pi/x\right)^{\pi x}$ is

A. $\exp(2\pi)$ B. $\exp(\pi)$ C. $\exp(\pi^2)$ D. NO

9. With the procedure

 reshape:=proc(L,n,m) local k; linalg[matrix](n,m,[seq(L[k],k=1..m*n)]); end:

   > **reshape([seq(k^2,k=1..9)],2,4);**

will produce a matrix with

A. 4 rows, 2 columns   B. 2 rows, 4 columns   C. 3 rows, 3 columns   D. NO

10. These multiple choice questions were

       A. Fair   B. Too hard   C. Too Easy   D. About right

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Part B: Maple Tasks.

Open a Maple session and record your work. <u>Name your file with your surname and initial (save it frequently)</u>. For example, Matt's file would be named **SkerrittM.mws**. *Remove things you do not wish marked.*

1. (a) Compute the factorization of the first 50 Mersenne numbers: $M(n) := 2^n - 1$.

   (b) Determine for which n the number is a Mersenne prime.

2. (a) Write code to evaluate $S_n := \sum_{k=0}^{n-1} F_{n-k} 10^k$ where F_n is the nth Fibonacci number

   for $1 \le n \le 20$.

   (b) What can you say about the behaviour of S_n for large n?

3. (a) Plot the following expression

$$x \sin\left(\frac{1}{x}\right) + y \sin\left(\frac{1}{y}\right)$$

   on the unit square $-1 \le x \le 1, -1 \le y \le 1$.
   Use enough points to get a realistic picture (Hint: look at **plot3d[option]**.)

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   (b) Describe qualitatively what you can see in the picture.

4. (a) Write Maple for the matrix below as a function of p and q:

$$A := \begin{bmatrix} p & q & 1-p-q \\ 1-p-q & p & q \\ q & 1-p-q & p \end{bmatrix}$$

   (b) By examining various numerical cases, conjecture the behaviour of the matrix A^n as $n \to \infty$ for $1 - p - q > 0$, $p > 0$, $q > 0$.