REFERENCES

MathResources Inc.

Enigma

J.M. Borwein and D.H. Bailey, *Mathematics by Experiment: Plausible Reasoning in the 21st Century,* A.K. Peters, 2nd expanded edition, **2008**

and with

R. Girgensohn, *Experimentation in Mathematics: Computational Paths to Discovery,* A.K. Peters, 2004. [Active CDs 2006]

D. Bailey, J. Borwein, N. Calkin, R. Girgensohn, R. Luke, and V. Moll, *Experimental Mathematics in Action*, A.K. Peters, 2007

J.M. Borwein, E.M. Rocha, and J.F. Rodrigues, *Communicating Mathematics in the Digital Era*, A.K. Peters **2008**

Jon Borwein and Keith Devlin, *The Computer as Crucible*, A.K. Peters **2008**

“The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.”

- J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.

December 4th 2008
Auckland

The past 60 years in Mathematics

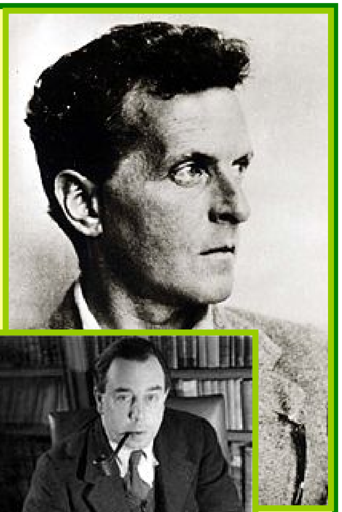
Jonathan Borwein, FRSC www.cs.dal.ca/~jborwein

Canada Research Chair in Collaborative Technology, Dalhousie 

 Laureate Professor, University of Newcastle

It's generally the way with progress that it looks much greater than it really is. (Ludwig Wittgenstein 1889-1951)

"whereof one cannot speak, thereof one must be silent"

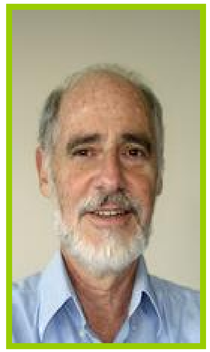


The world will change. It will probably change for the better. It won't seem better to me (J. B. Priestley, 1894-1984)

There was no respect for youth when I was young, and now that I am old, there is no respect for age. I missed it coming and going.

Revised 03/12/2008

A Talk in Honour of Three of "Us"


chronophage

- **Paul Hafner** *Ordered fields, bilinear forms, computations with groups and graphs*
- **Joel Schiff** *Complex analysis, potential theory, arithmetic Fourier transform*
- **David Smith** *Commutative ring theory*

Abstract of Presentation

I shall explore my **personal mathematical experiences** over the past six decades as a way of investigating idiosyncratically:

- what's changed and what remains much the same
- what has been gained and what has been lost

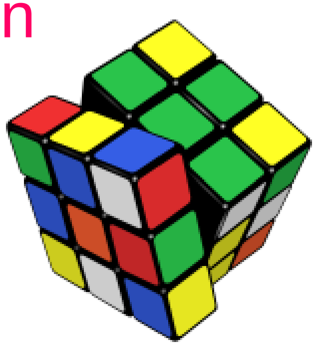
- I'll do this in 'very roughly' decade-long tranches.

"Knowing things is very 20th century. You just need to be able to find things." (Danny Hillis)
- how Google has changed the way we think: Achenblog, July 1 2008
(**1925** von Neumann claimed to 'know' 25% of extant mathematics)

Eight Results We Did Not Know

- Independence of Continuum Hypothesis (Cohn 1963)
- Luzin Conjecture: a.e. Fourier convergence in L^2 (Carleson 1966)
- Four Colour Theorem (Appel-Haken, 1976-1997)
- Classification of Finite Simple Groups (Feit-Thompson 1955-1995 maybe?)
- Fermat's Last Theorem (Wiles-Taylor, 1993-94)
- Poincaré Conjecture (Hamilton-Perleman 2004, …)
- Primes in long arithmetic progressions (Green-Tao 2008)
- The Most Striking Result in Your Own Area

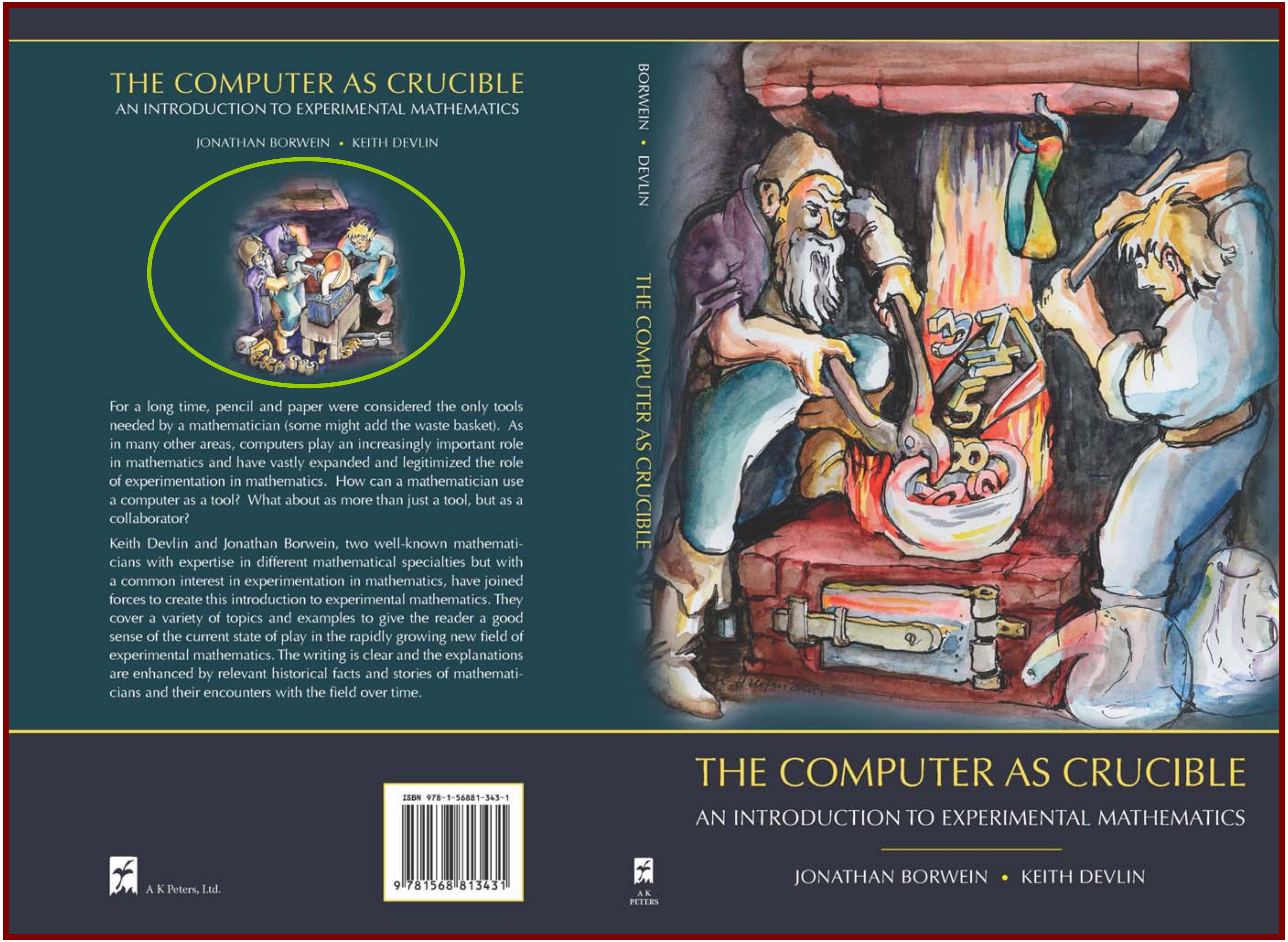
Wikipedia



"***The most important aspect in solving a mathematical problem is the conviction of what is the true result.*** *Then it took 2 or 3 years using the techniques that had been developed during the past 20 years or so."* (Lennart Carleson, 1966 ICM)

"The Future"

AK Peters, 2008



1948 to 1962

I. 1948 to 1962: From South Africa to the UK

II. 1963 to 1970: Ontario

III. 1971 to 1982: Oxford, Halifax, Pittsburgh

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V. 1994 to 2000: Vancouver

VI. 2001 to 2008: From BC to **NS** to **NSW**

For each section I have tried to build a brief list of salient events comprising both pros and cons

1948 to 1962

1948 DB starts Univ. Col. PhD with Bosanquet (Roth, Davenport)

- read every `pertinent' paper then start research (a crazy recipe now)
- DB 5 carbon copies and kept original (my DPhil follows)
- Dad's *Underwood* had six math symbols: $\lambda, \mu, \tau, \sum, \int, \lim$

1944 Radar Borwein

1949 CJM starts: Einstein + galaxy of stars in issue 1

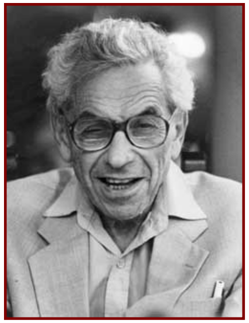
194x Modern computing starts

- well sort of (most great 20C algorithms found before 1970)

1950 Dad goes to StA (6 ads) so Mum does ('one body problem')

1948/54 15 days from Southampton to Capetown

- DB plays Nepune (3rd crossing of Equator)



1951 DB & JMB meet Erdös (as DB is most junior in dept)

"What is ε_3?" **1961 CROSS-CURTAIN POSTCARDS** "$\varepsilon_3 = ♀$"

JMB's DPhil: a Pre IBM-Selectric Sample

Such theorems are generally called Fritz John type conditions.

[2] Theorem. Suppose x is a weak (local) minimum with respect to S for (Q). Suppose f, g are compactly differentiable at x. Then there exist $p^+ \in S^+$, $q^+ \in B^+$, not both zero, such that

$$p^+ f'(x) + q^+ g'(x) \in P^+(C,x) \qquad q^+(g'(x)) = 0.$$

Proof: Let $M = \left\{(y,z) \mid \exists\, h \in T(C,x) \text{ with } (f'(x)(h) \leq_S y, \; (g'(x))(h) \leq_B z \right\}$

$$N = \left\{(y,z) \mid y \in -S,\; z \in -B\right\}.$$

M and N are closed convex sets and $N^o \neq \emptyset$.

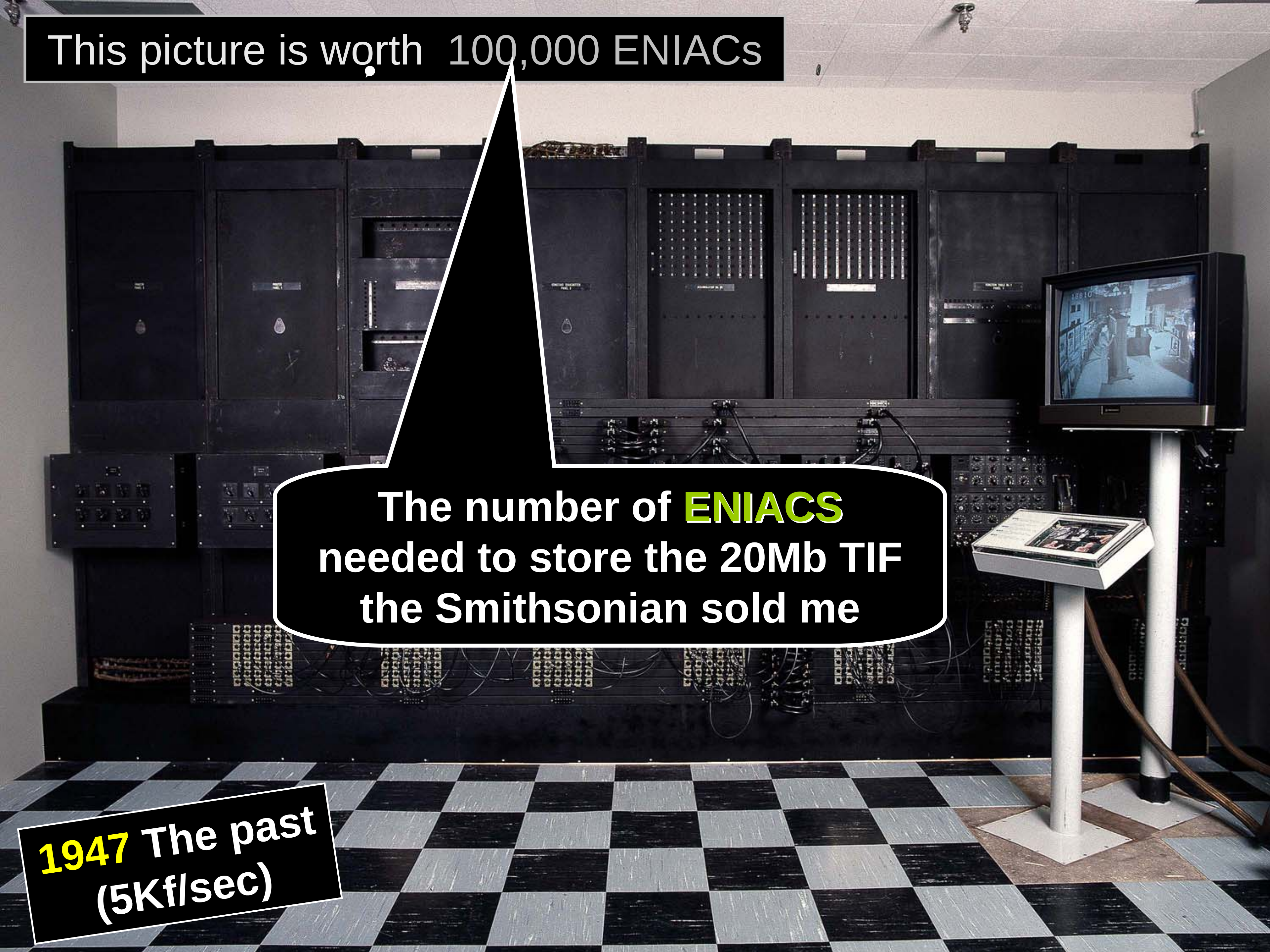
Suppose that $M \cap N^o \neq \emptyset$. There then exists $h \in T(C,x)$ with $(f'(x))(h) \in -S^o$ and $(g'(x))(h) + g(x) \in -B^o$.

Let $h_n = \lambda_n(x_n - x) \to h$ where $x_n \in C$, $x_n \to x$, $\lambda_n \geq 0$. By the definition of the compact derivative

$$(1) \quad \lambda_n\left[f(x_n) - f(x)\right] = \frac{f(x + \lambda_n^{-1} h_n) - f(x)}{\lambda_n^{-1}} \to (f'(x))(h) \in -S^o .$$

Canadian Journal of Mathematics Volume One

Imagine a similar launch in 2009


This picture is worth 100,000 ENIACs
The number of ENIACS needed to store the 20Mb TIF the Smithsonian sold me
1947 The past (5Kf/sec)

1963 to 1970

I. **1948 to 1962:** From **South Africa** to the **UK**

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The budget of Dalhousie in 1975 (Hicks was VC) was larger than NS in 1955 (when Hicks was Premier)

1963 to 1970

1963 DB hired to make Western Ontario a research dept

- UWO now one of top 5 University's in Canada

1967 Astronomy professor's wife could only be an instructor

- 'anti-nepotism' clauses

1960 -1990? Promotion routinely refused on anti-semitic grounds

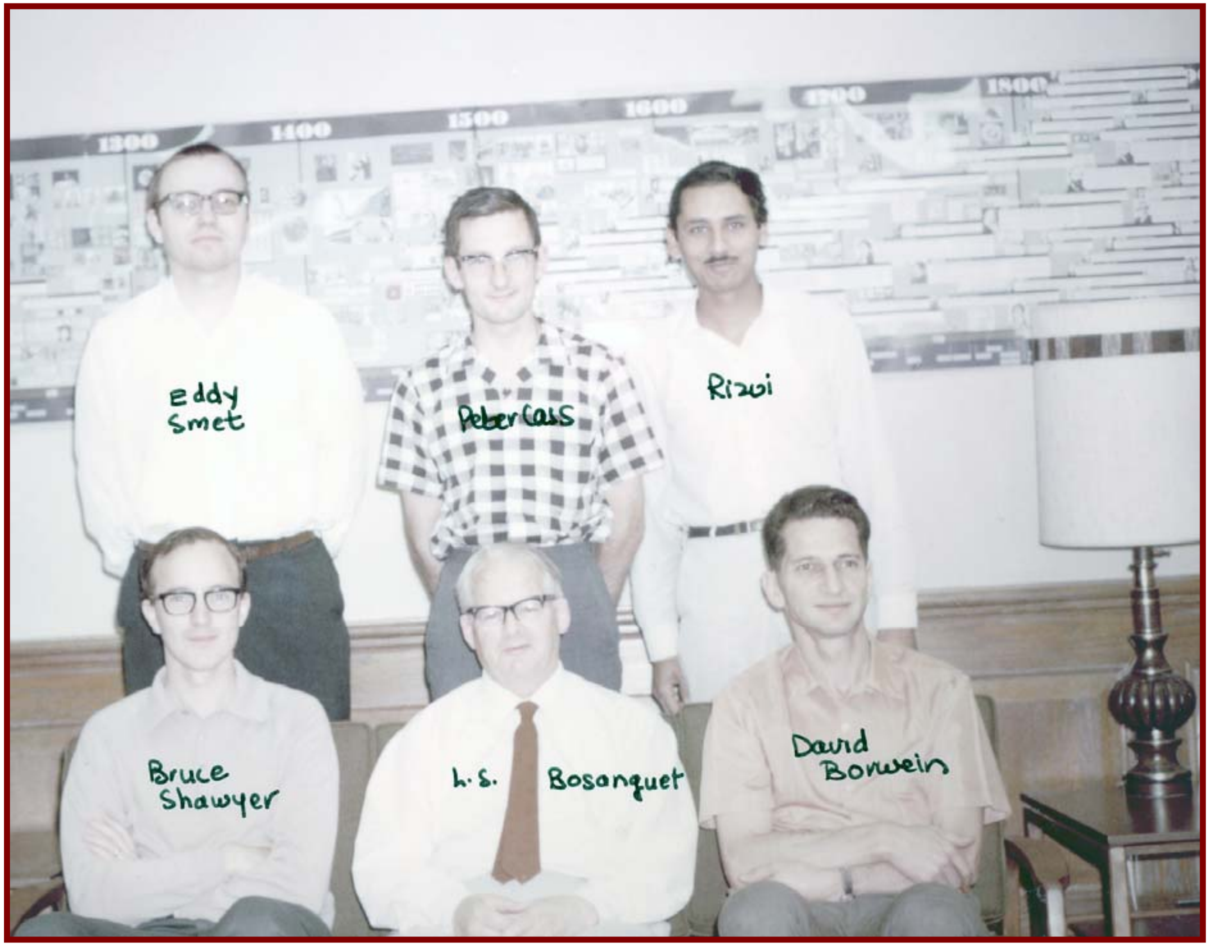
- slide rules, mimeographs and airmail letters prevailed

1963- 1970 Sputnik briefly met Bourbaki

- English speaking academic community more than doubled ('massification')

1997 "Thank God it's so different today!" Mr. Wiesel said. "Today, there are Jewish presidents at Princeton, Harvard, Yale and Northwestern. Something happened in America. Of course we had to pay a price: the Holocaust. But the Holocaust has changed America."

1963 to 1970



DB relaxing at home

1968 My first Functional Analysis course from LS Bosanquet
- a student of GH Hardy (and 'author' of *Divergent Series*)

1965 Moore's Law & Implications

"The complexity for minimum component costs has increased at a rate of roughly a factor of two per year ...

- now taken as "every 18 months to 2 years"

Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years. That means by 1975, the number of components per integrated circuit for minimum cost will be 65,000. I believe that such a large circuit can be built on a single wafer.

Gordon Moore (**Intel**) "Cramming more components onto Electronic Circuits", *Electronics Magazine* 19 April 1965

Unprecedented and **still** expected to continue for 10-20 years

1971 to 1982

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Oxford Univ. only lost members on City Council in 1974

- and the right to hang students?

1971 to 1982

1966-74 ties & gowns disappear in Oxford but 'fiancées' arrive

- **Xerox** (1958-61) arrives in public
  - only allowed one copy
- So do **TI** calculators (CASIO blew it) and "Pong"
  - slide rule is in Smithsonian within 5 years

1972 "Trunk calls" were expensive and difficult
  - to call home we stayed a night at Paddington hotel

1978- Personal computing and $\TeX$ (frozen in 1988) arrive
  - how Dave Bailey did not make his first $100 million

1979 Carnegie-Mellon put all pre-1948 material on micro-fiche and sold originals

Logarithms took root as fast as TI as Gary Tee can attest

1983 to 1993

I. **1948 to 1962:** From **South Africa** to the **UK**

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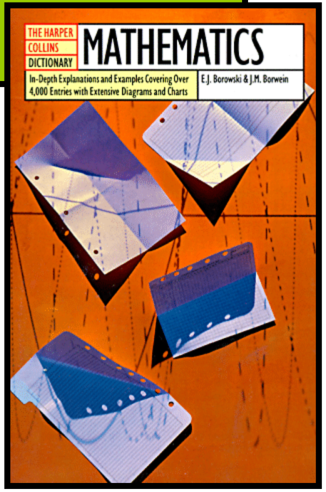
In 1989 we had to shut down Dalhousie's main computer to compress a 16Mb **ISC** book file: now the **iPhone** has 8Gb

1983 to 1993



1983: I design my best code for a class

- the “idiot pivoter”: to make teaching **simplex** tolerable

1985 - contracted to author Collins dictionary

- **1985** given “electronic and musical rights”
- blessed six week turn around as we edited UK↔NS
- **1988** 1st book set from disk in Europe; **FAX** finds me here
- **1995** we asserted digital rights to Harper who replied “much as we hate to agree.” We founded Math Resources Inc.

- Faculty clubs started to die as life-styles changed
- Honours classes began to disappear and curiosity-driven research funds shrank

The state of the art LP package (CPLEX) still does not **prove** anything

- one reason Hales had problems publishing his solution of Kepler’s problem

1983 to 1993

1991-1993 I Set up CECM (now PBB's IRMACS)

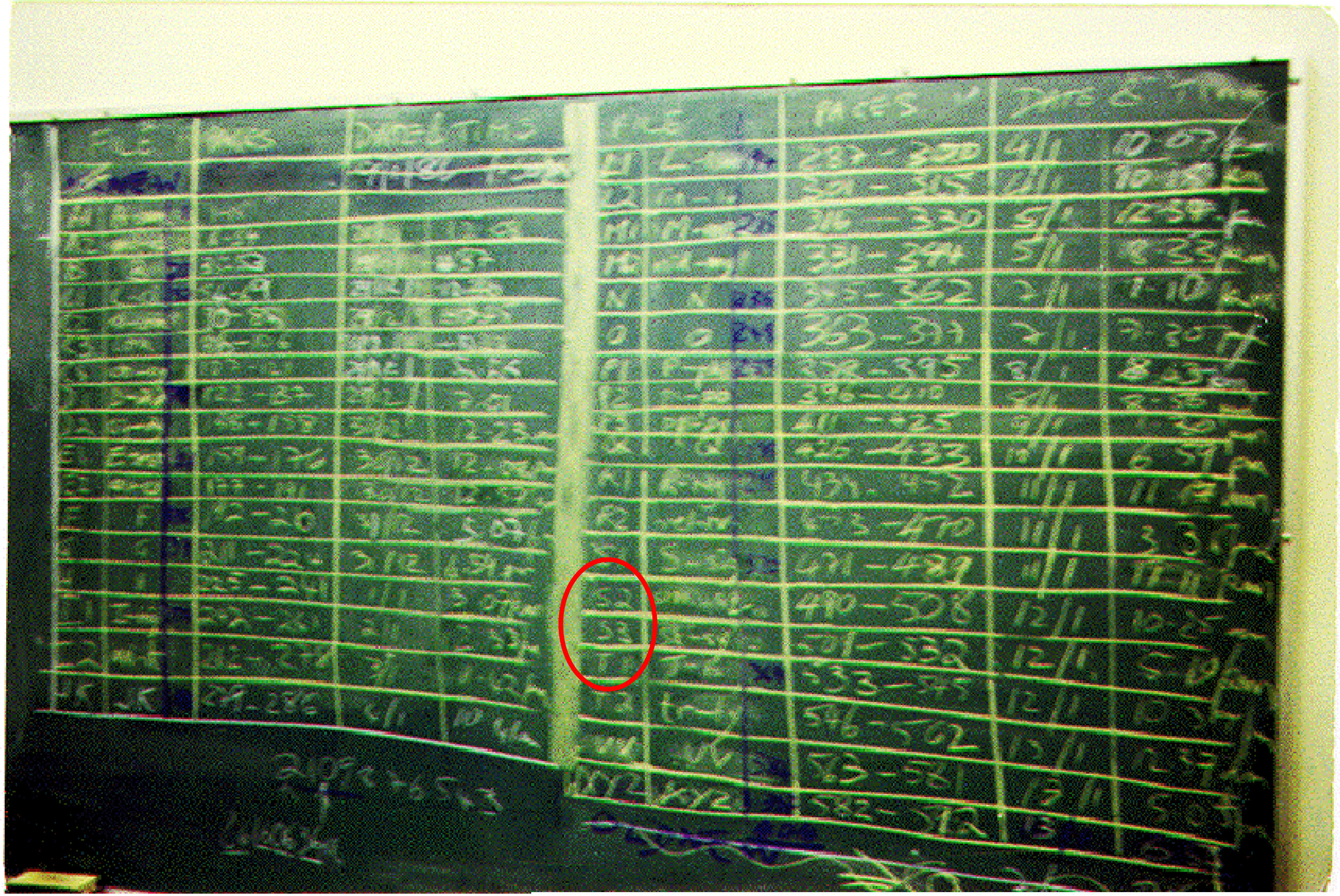
- **SUN** work stations were work horses
  – **SGI** Indy's were cool and cost effective
  – IT flowed out of central computing into departments

1993 WWW arrived democratizing mathematics

  – for better and worse (anti-social networking)
  – we were already serving data through Archie, ….

"A centre of excellence is, by definition, a place where second class people may perform first class work."
from J.M. Thomas, **Michael Faraday - and the Royal Institution**, 1991.

A 1985 Relational Silicon Database



Controlling: **3 Apple+s**, a **brain-dead Lisa**, **MacWrite,** and **dozens of 5” floppies**

1994 to 2000

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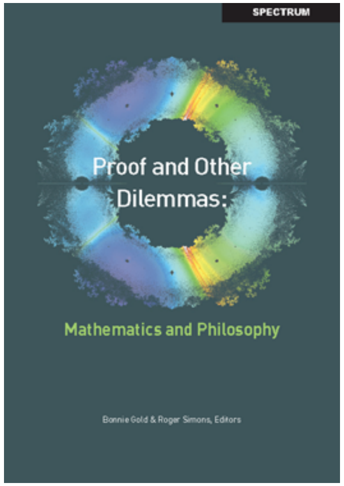
My morale has never been higher than since I stopped asking for grants to keep my lab going, Robert Pollack, Columbia, speaking on "the crisis in scientific morale", September 19, 1996 at GWU symposium *Science in Crisis at the Millennium*.

1994 to 2000

1994 Java arrived & St Andrews history MacTutor (MT) opened

1995 we built the **Organic Math Project**

still largely unmatched collection of 30 “activated” mathematical articles (I am amazed at its resilience)



- Hopes for the “open frontier” were very high
  - read Lawrence Lessing on IP in 1995 and 2005
- Advanced Mathematical Computing became possible
  - *Maple*, *Mathematica* and MATLAB reached near maturity

1996 MathSciNet went public (see following pages: in both cases I asked for and got details within the day)

> “*The most important intellectual event of 1997 took place in a warehouse outside Seattle*”

1994 to 2004

From Edmund Robertson: “John O'Connor and I won the European Academic Software Award in 1994 with **MacTutor** and we demonstrated the first version of the web Archive in Heidelberg as part of our submission. So its been running now for about 14 years.

The average number of hits per day over the last seven days was **316,245 per day**. This number still amazes me! About **4.5 gigabytes** of data are transferred per day. These numbers have been pretty constant now for at least the last 5 years with between 300,000 and 400,000 hits per day. There are about **2000 biographies** ... I'm sure that is more than enough information.

On a personal note: the first university course I attended was 'Analysis' given by your father in 1961 at St Andrews. I retired one week ago - how quickly the years go by.

A final thought - I'd love to put a version of your talk on the archive.”

MacTutor, along with the **Math Genealogy project**, is a labour of love of enormous value and an uncertain future

1994 to 2000

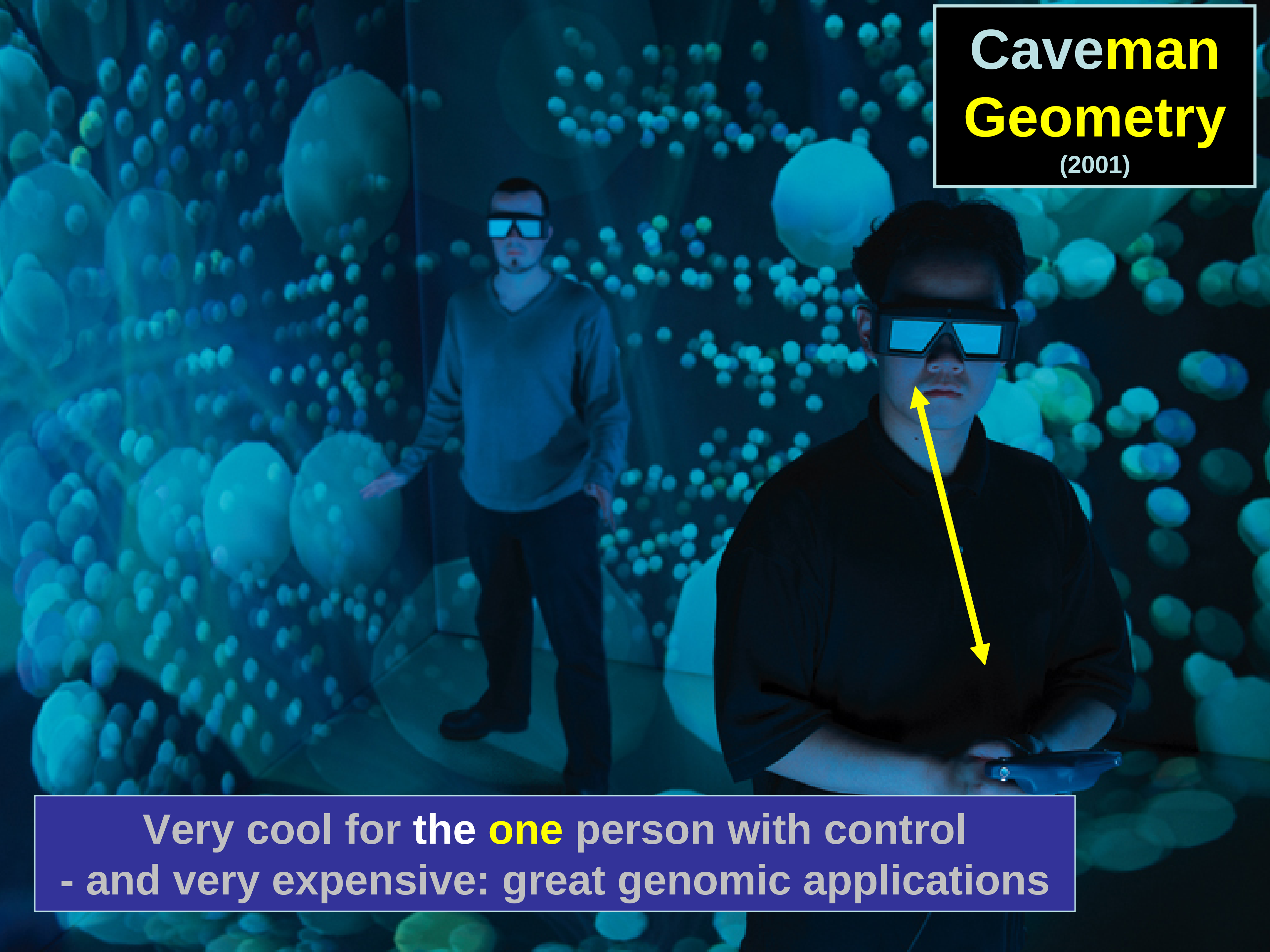
From John Ewing: "MathSciNet first went live to the public in January, 1996, after a frantic development cycle in a matter of months.

It began adding links to original articles in **1997**, and now has more than 1 million links. (More than half the journal articles listed in MathSciNet have links, which means **well more than half the journal literature is online**.)

It began adding reference lists to a selection of journals (now about **420**) in **1999**, and this forms the basis for the **mathematics citation database**.

Math Reviews has done **author identification** for the past 68 years, uniquely identifying (by hand) every author of every article. (There are now more than **512,000 authors** so indexed!). The results of this effort are now contained on an author profile page (see your own at http://www.ams.org/mathscinet/search/author.html?mrauthid=39830)"

For most purposes AMS now provides a much more reliable citation barometer than ISI or Google: but less so for more applied areas.


Caveman Geometry
(2001)
Very cool for the one person with control
- and very expensive: great genomic applications

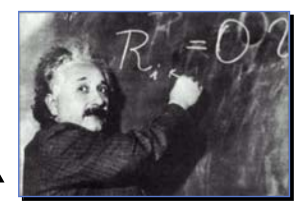
The Changing Research Landscape: a new triangle

Computational

(dry science)

Experimental

(wet science)



Theoretical

(thought experiments)

2001 to 2008

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Concentration of ownership of Mathematics Journals by a few big publishers has accelerated over the decade

2001 to 2008

1948 A good PhD was **original** but might now take few seconds in MATLAB

2008 A good PhD must be a **discovery*** ("independent, reliable and rational")

– but may well not be original

2007 The reference book market is collapsing

– publishers still do not understand software

- Time is replacing space as a distance metric

2008 Maths lags badly behind most disciplines in technology use

– **the Access Grid**

– **web services and databases**

– **symbolic computation, visualization and more**

- That said, mathematics is more highly respected than at any time in my lifetime (before the crash?)

* M. GiaQuinto, *Visual Thinking in Mathematics*, OUP 2007 (Reader at UC London)

Each Mathematics ISI Citation is worth 15 in Physics and 80 in Medical Science

My Lab in Halifax

240 cpu Glooscap at Dal

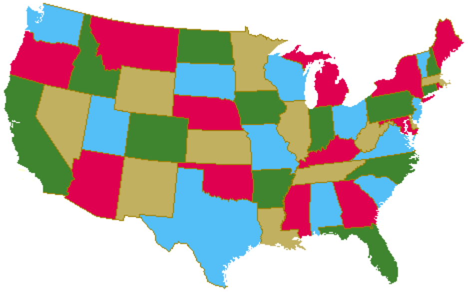
Dalhousie Distributed Research Institute and Virtual Environment

D-Drive's Nova Scotia location lends us unusual freedom when interacting globally. Many cities around the world are close enough in a chronological sense to comfortably accommodate real-time collaboration.

"It says it's sick of doing things like inventories and payrolls, and it wants to make some break-throughs in astrophysics."

The Current End 2001 to 2008

A cross-section of illustrations follows:



- The Access Grid
- Web services and databases
- Symbolic computation, visualization and much more

We should both be exploiting and be worrying about their use since our subject relies more on the reliability of its literature than does any other science

REFERENCES

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“The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.”

- J. Hadamard quoted at length in E. Borel, *Lecons sur la theorie des fonctions*, 1928.

Dalhousie Distributed Research Institute and Virtual Environment

Coast to Coast ('C2C') Seminar

2008: will focus on PhD presentations

Chile has now joined

Lead partners:

Dalhousie D-Drive – Halifax Nova Scotia

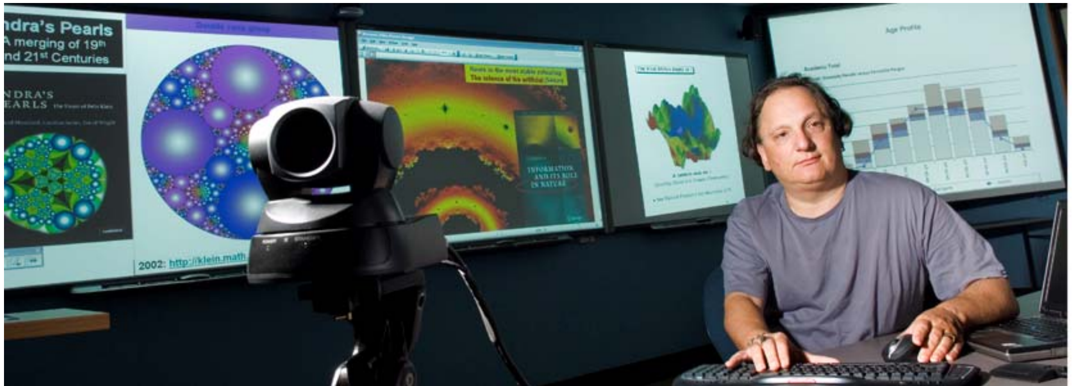
IRMACS – Burnaby, British Columbia

Other Participants so far include:

University of British Columbia, University of Alberta, University of Alberta, University of Saskatchewan, Lethbridge University, Acadia University, MUN, Mt Allison, St Francis Xavier University, University of Western Michigan, MathResources Inc, University of North Carolina, …

Tuesdays 3:30pm (Atlantic) 11.30am (Pacific)

✓Chapter in Communicating Mathematics in the Digital Era (AK Peters, Sept 2008)



I could be in **Newcastle AG**

CARMA is coming

Computer **A**ssisted **R**esearch **M**aths and its **A**pplications

The Experience

Fully interactive multi-way audio and video

Given good bandwidth audio is much harder (if you rehearse)

The closest thing to being in the same room

Shared Desktop for viewing presentations or sharing software

"Solving Checkers"

Speaker in Edmonton

Audience in Vancouver

April 2007 **Checkers solved**

Science: one of top 10 break-throughs of 2007

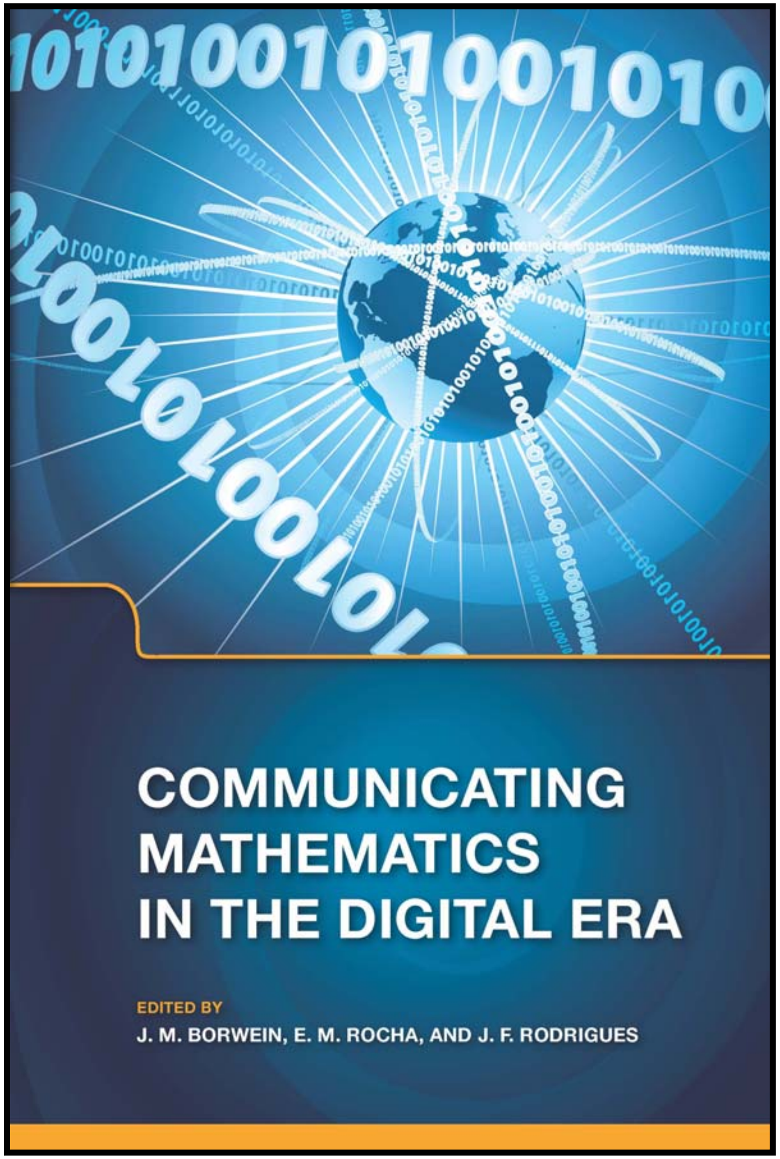
2006: **Poincaré Conjecture** top breakthrough of year

2006 ICM Satellite Meeting Collection

AKPeters, October 2008

"The digital era has dramatically changed the ways that researchers search, produce, publish, and disseminate their scientific work. These processes are still rapidly evolving due to improvements in information science, new achievements in computer science technologies, and initiatives such as DML and open access journals, digitization projects, scientific reference catalogs, and digital repositories.

These changes have prompted many mathematicians to play an active part in the developments of the digital era, and have led mathematicians to promote and discuss new ideas with colleagues from other fields, such as technology developers and publishers. This book is a collection of contributions by key leaders in the field, offering the paradigms and mechanisms for producing, searching, and exploiting scientific and technical scholarship in mathematics in the digital era."



IBM BlueGene/L at LANL

System
(64 cabinets, 64x32x32)

IBM Computer Achieves Petaflop Performance

6/9/2008

A National Nuclear Security Administration (NNSA) supercomputer has achieved an operational rate of 1,000 trillion calculations per second, or 1 petaflop, making the Roadrunner -- which the NNSA commissioned IBM Corp. to build in 2006 for around $130 million -- the world's fastest computer, the agency announced today.

2.8/5.6 GF/s
4 MB

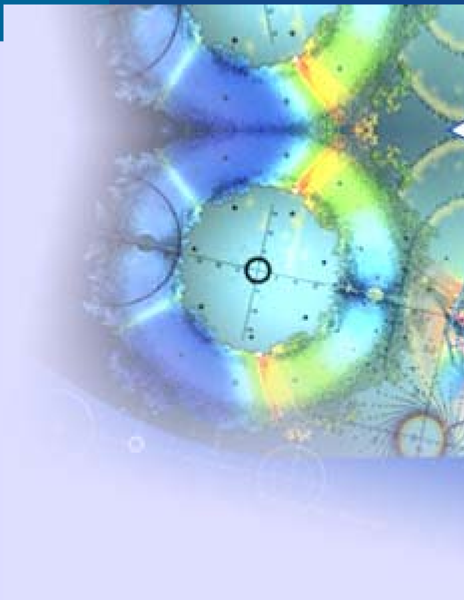
0.5 GB DDR

2^{17} cpu's: Oct 2007 ran Linpack benchmark at over **596 Tflop /sec**

(5 x Canada or 8 x Oz)

The future
2005-2010

Jon Borwein's Mathematics Portal



The following is a list of useful math tools. The distinction between categories is somewhat arbitrary.

Utilities (General)

1. The On-Line Encyclopedia of Integer Sequences
2. ISC2.0: The Inverse Symbolic Calculator
3. 3D Function Grapher
4. Julia and Mandelbrot Set Explorer
5. The KnotPlot Site

Utilities (Special)

6. EZ Face : Evaluation of Euler Sums and Multiple Zeta Values
7. GraPHedron: Automated and Computer Assisted Conjectures in Graph Theory
8. Embree-Trefethen-Wright Pseudospectra and Eigenproblems
9. Symbolic and Numeric Convex Analysis Tools

Reference

10. NIST Digital Library of Mathematical Functions(X)
11. Experimental Mathematics Website
12. Numbers, Constants, and Computation
13. Numbers: the Competition
14. The Prime Pages

Haptics and Light Paths

D-DRIVE Doug a haptic mascot

Drive
Dalhousie Distributed Research Institute and Virtual Environment

Haptic Devices extend the world of I/O into the tangible and tactile

To test latency issues …

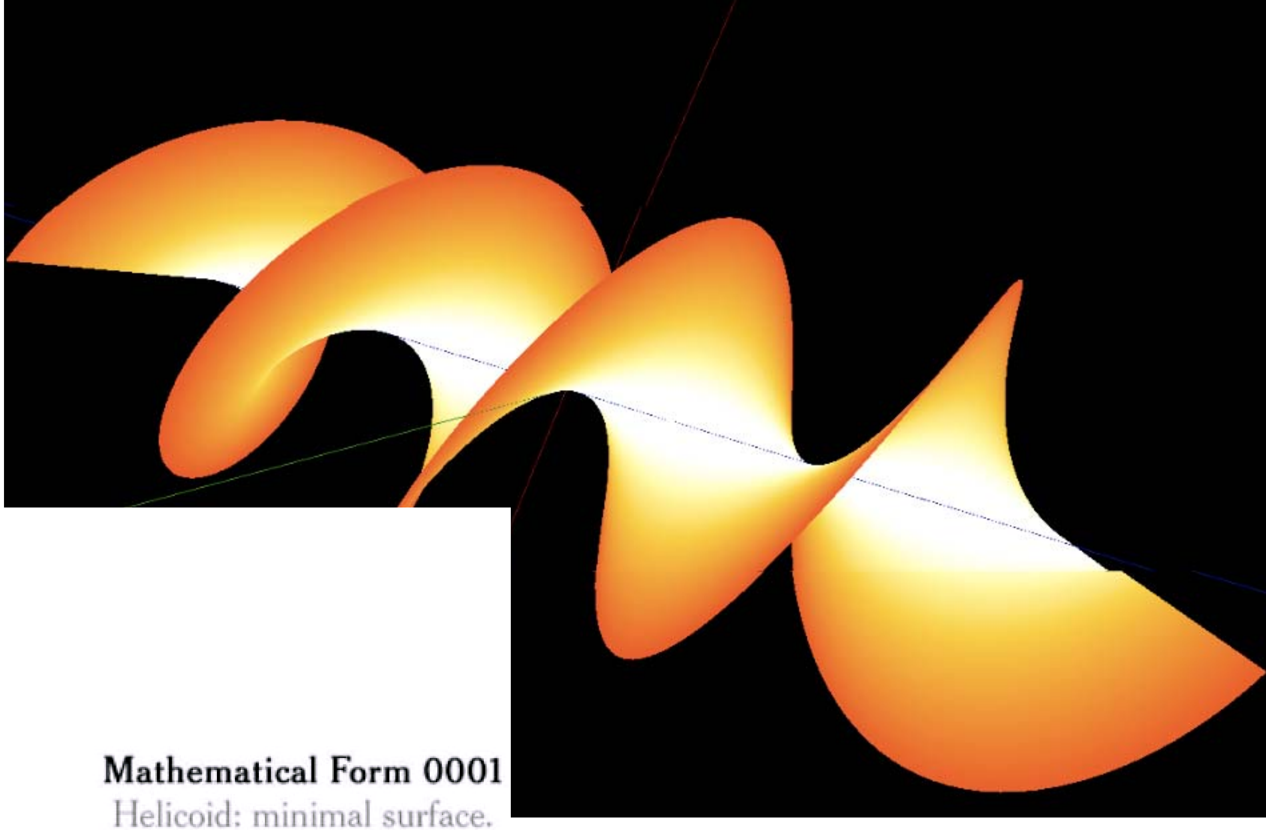
Links multiple devices so two or more users may interact at a distance (**BC/NS Demo** April 06)

- in Museums, **Aware Homes**, elsewhere
- Kinesiology, Surgery, Music, Art …

Sensable's **Phantom Omni**

Cost effective 3D visualization in 2007

19th C model plus recent photograph and 21st C rendition



Hiroshi Sugimoto for The New York Times

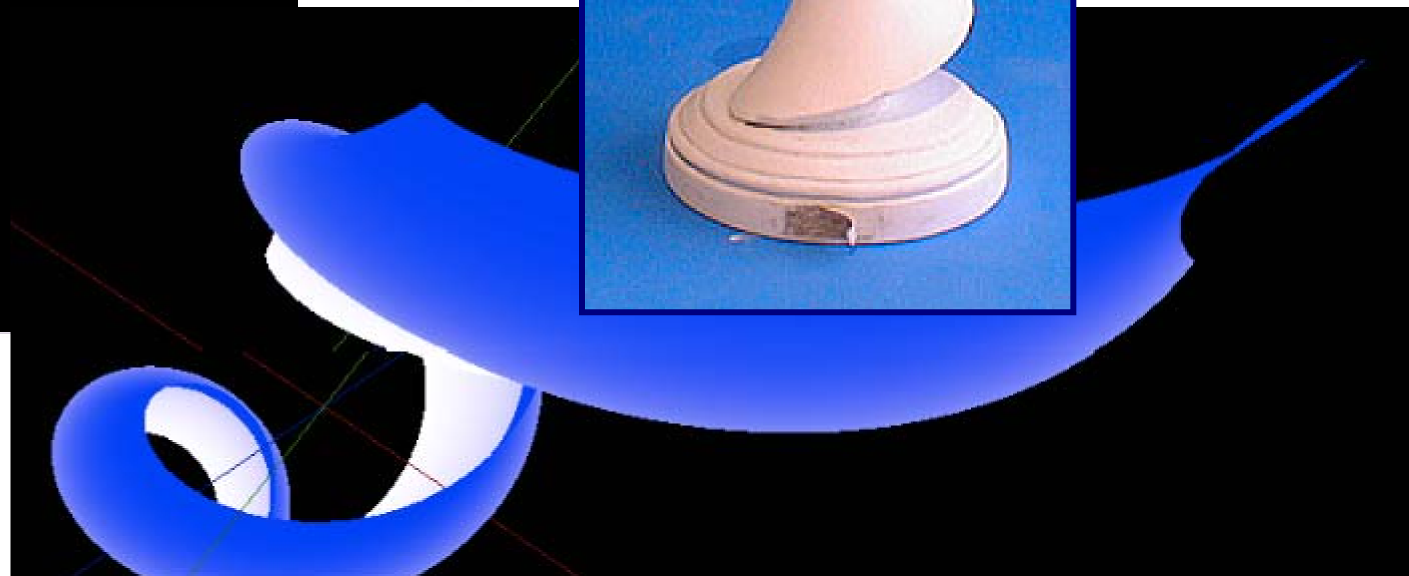
Mathematical Form 0001

Helicoid: minimal surface.

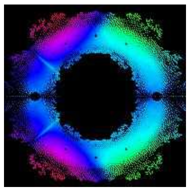
$$x = a \sinh v \cos u$$
$$y = a \sinh v \sin u$$
$$z = au$$
$$(0 \leq u < 2\pi, -\infty < v < \infty)$$

19th C Plaster Model
Kline and Schwartz

Hiroshi Sugimoto



Content Must Dominate Form



Jonathan Borwein, Dalhousie University
Mathematical Visualization

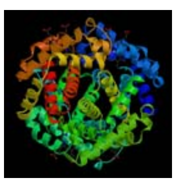
High Quality Presentations

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems

Peter Borwein, IRMACS
The Riemann Hypothesis

"No one explains chalk"

Jonathan Schaeffer, University of Alberta
Solving Checkers



Arvind Gupta, MITACS
The Protein Folding Problem

Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants

Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes

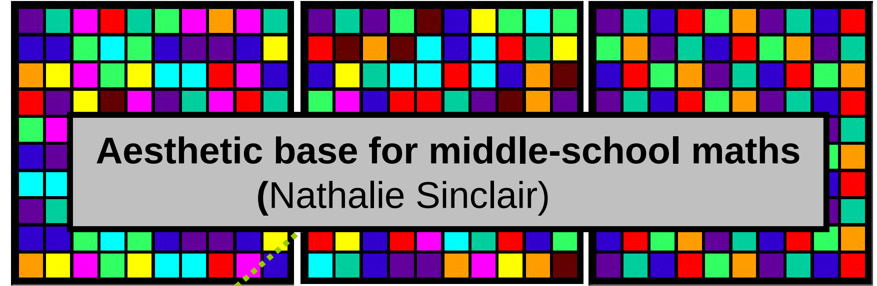
Future Libraries will include very complex objects

New Ways of Seeing Math

- **The Colour Calculator**
  – numbers as pictures
- **The Inverse Calculator**
  – numbers go in and symbols come out
- **The Top Ten Numbers Website**

- All at http://ddrive.cs.dal.ca/~isc/portal

A Colour and an Inverse Calculator (1995 & 2007)

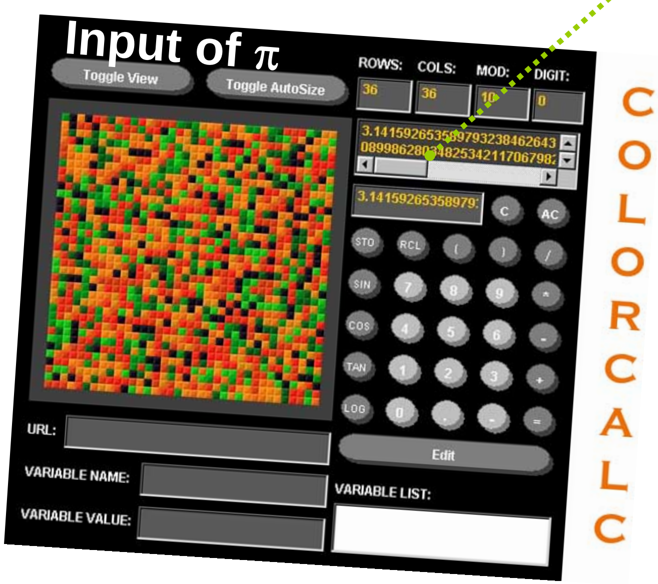


Archimedes: 223/71 < π < 22/7

Inverse Symbolic Computation

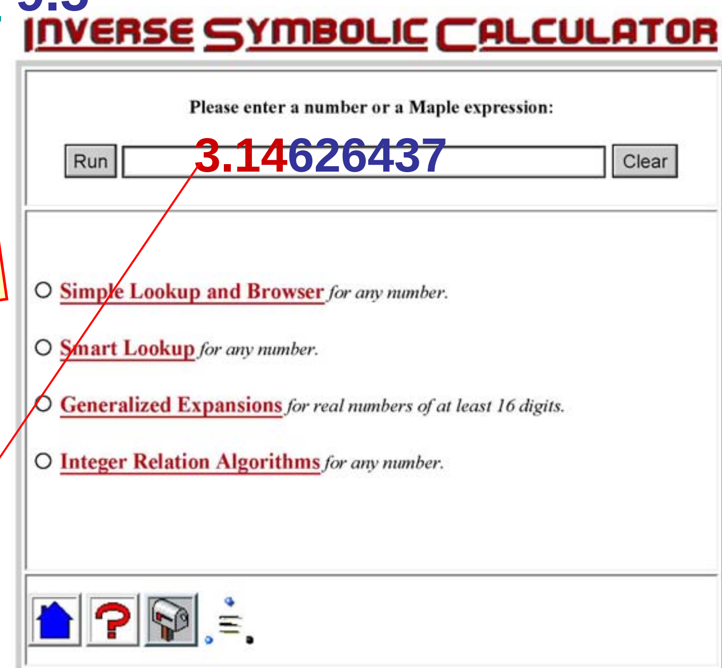
Inferring mathematical structure from numerical data

- Mixes *large table lookup*, integer relation methods and intelligent preprocessing – needs ***micro-parallelism***
- It faces the "curse of exponentiality"
- Implemented as **identify** in Maple **9.5**



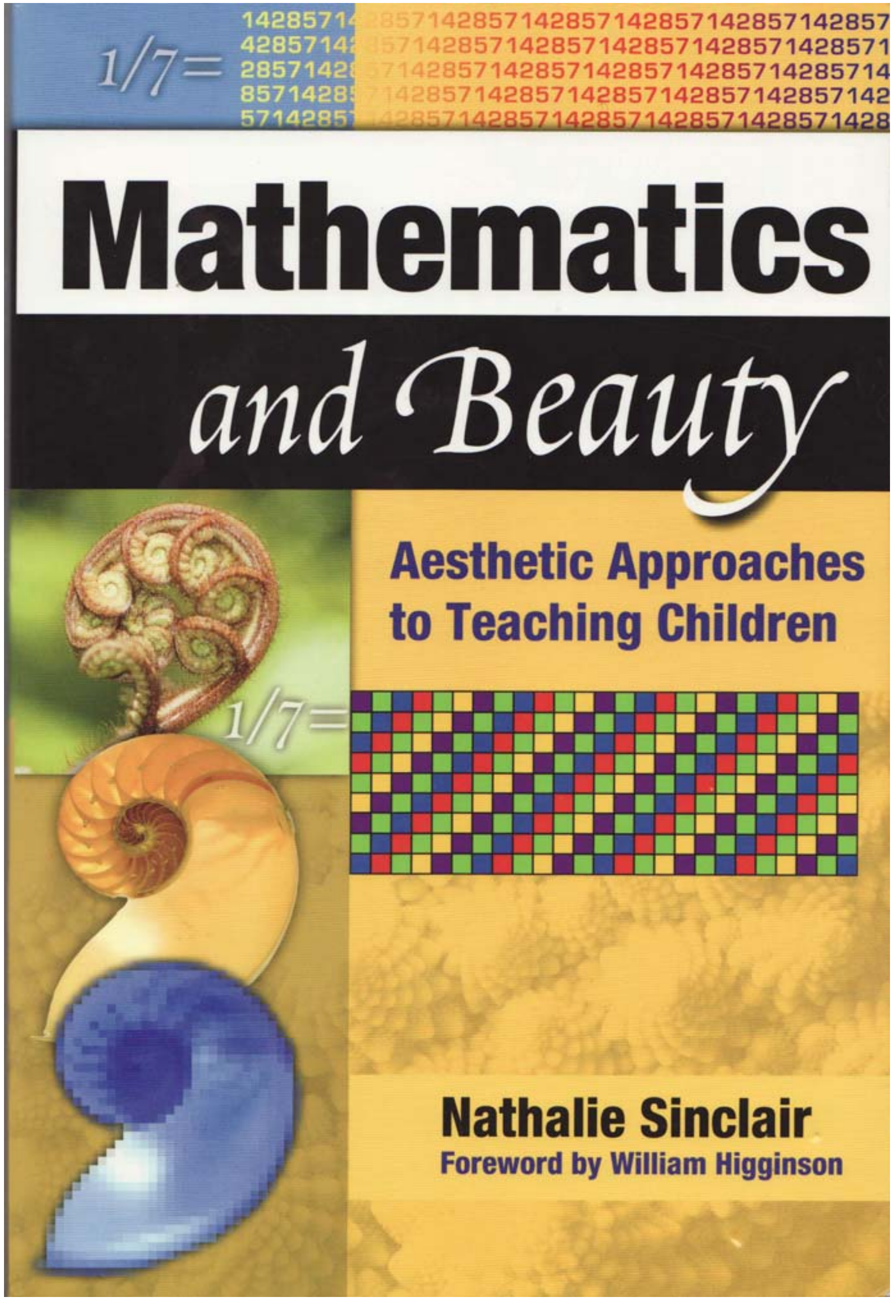
identify(sqrt(2.)+sqrt(3.))

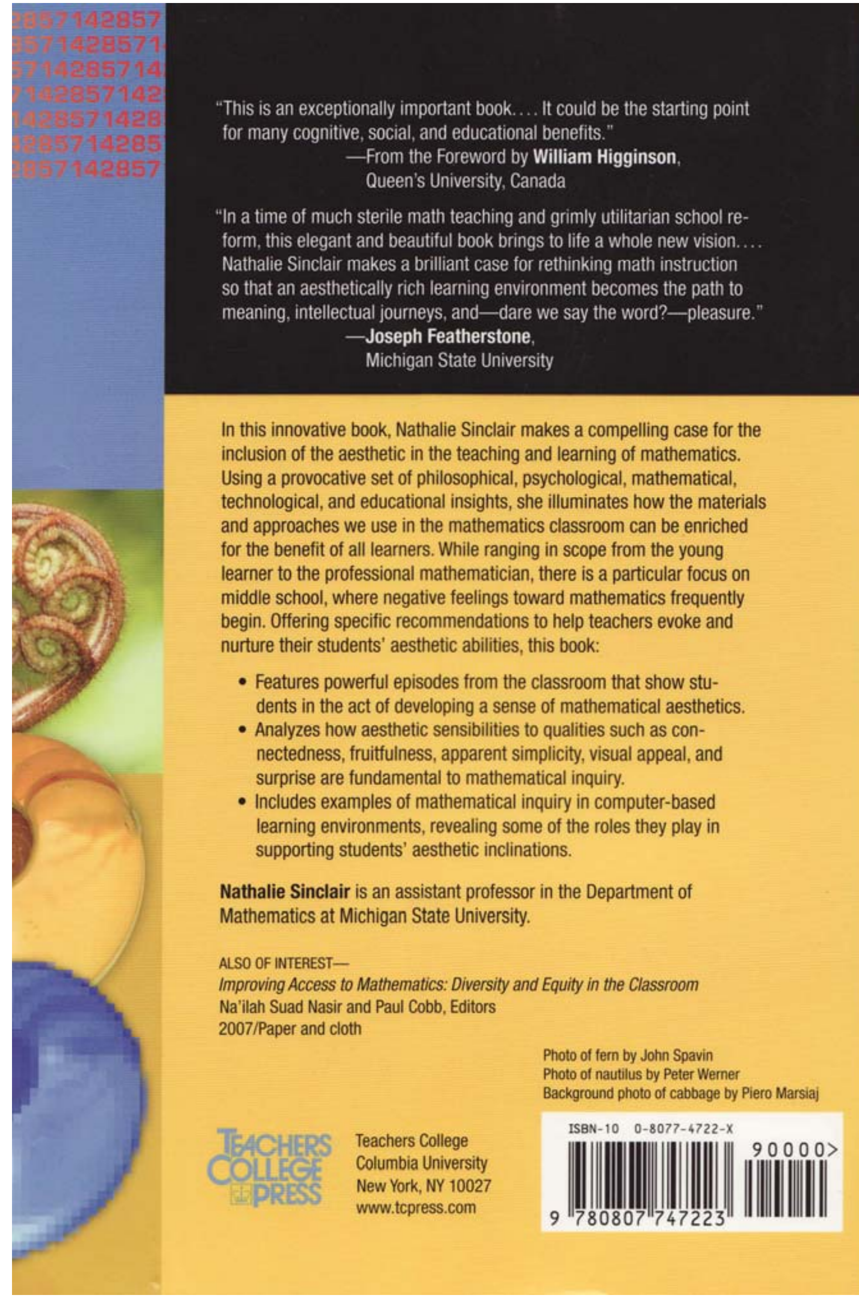
$$\sqrt{2} + \sqrt{3}$$



Expressions that are **not** numeric like ln(Pi*sqrt(2)) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.

Mathematics and Beauty 2006





The Inverse Symbolic Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

iDrive NSERC CRSNG Maplesoft

The ISC in Action

isc+ inverse symbolic calculator

Standard lookup results for **12.587886229548403854**

exp(1)+Pi^2

ISC [The original ISC]

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

3.146264370 Try it!

19.99909998 Try it!

ISC [The original ISC]

The Dev Team: Nathan Singer, Andrew Shouldice, Lingyun Ye, Tomas Daske, Peter Dobcsanyi, Dante Manna, O-Yeat Chan, Jon Borwein

The ISC presently accepts either floating point expressions or correct Maple syntax as input. However, for Maple syntax requiring too long for evaluation, a timeout has been implemented.

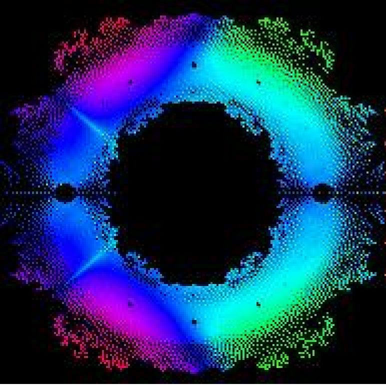
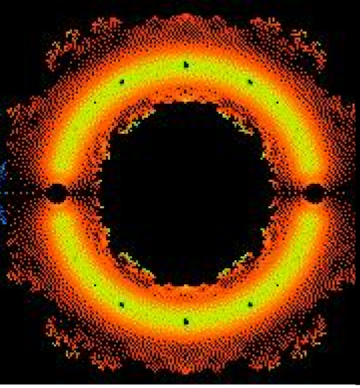
Visit

[Jon Borwein's Webpage]

[David Bailey's Webpage]

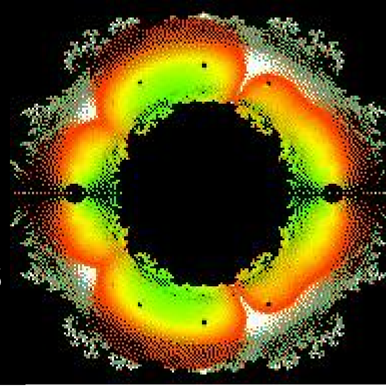
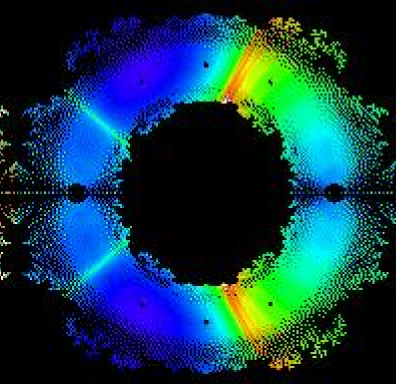
[Math Resources Portal]

- ISC+ runs on Glooscap
- Less lookup & more algorithms than 1995

Roots of Zeros

What you draw is what you see ("visible structures in number theory")

Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of x with coefficients 1 and -1 to degree 18

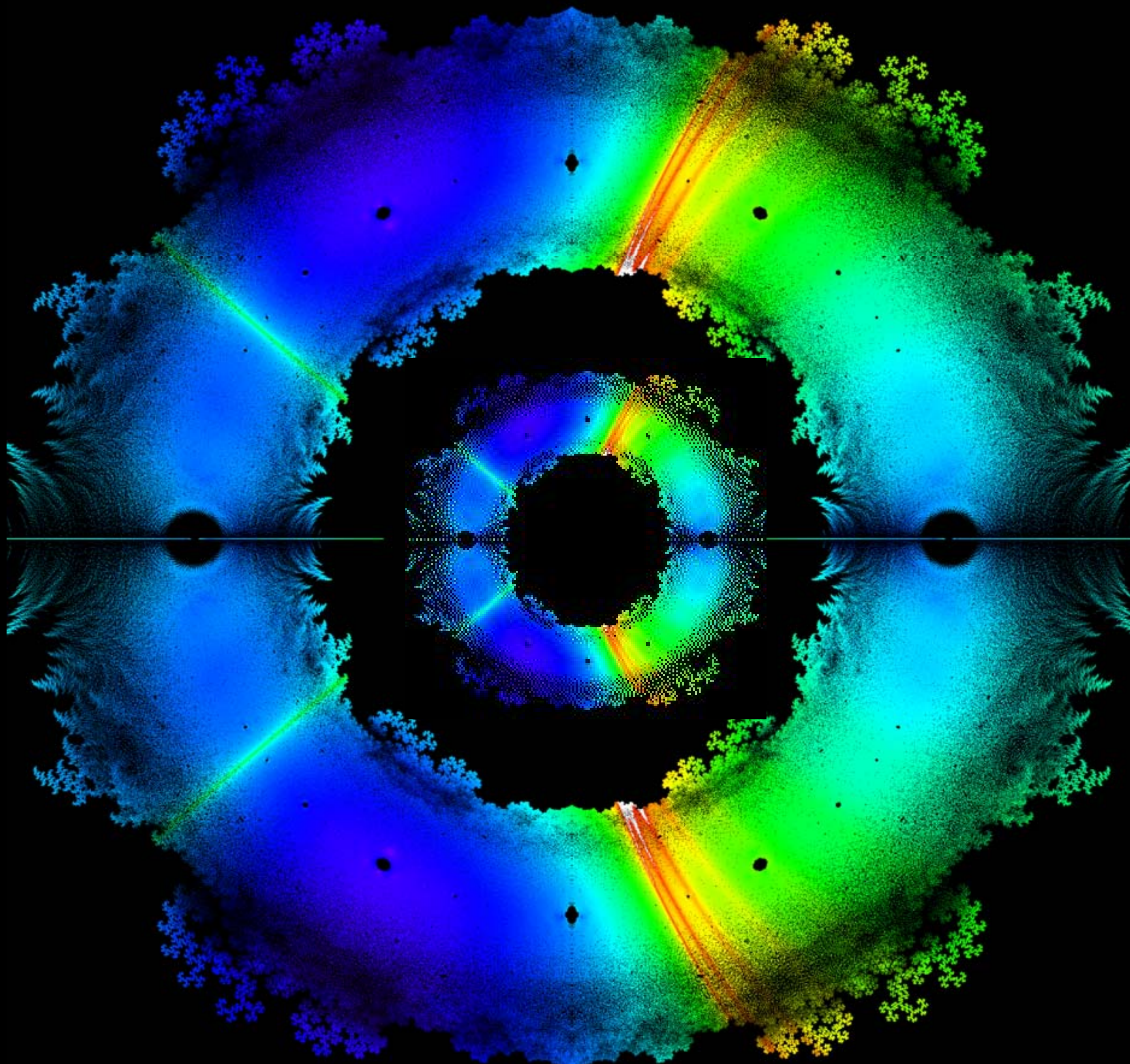
Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. **The color scale represents a normalized sensitivity** to the range of values; red is insensitive to violet which is strongly sensitive.

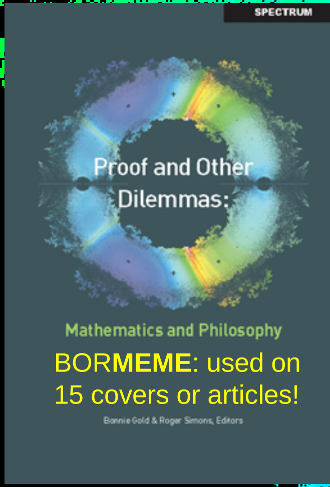
- All zeros are pictured (at **3600 dpi**)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $\mathbf{x^9}$ term
- **The white and orange striations are not understood**

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!" Greg Chaitin, Interview, 2000.

The TIFF on VARIOUS SCALES





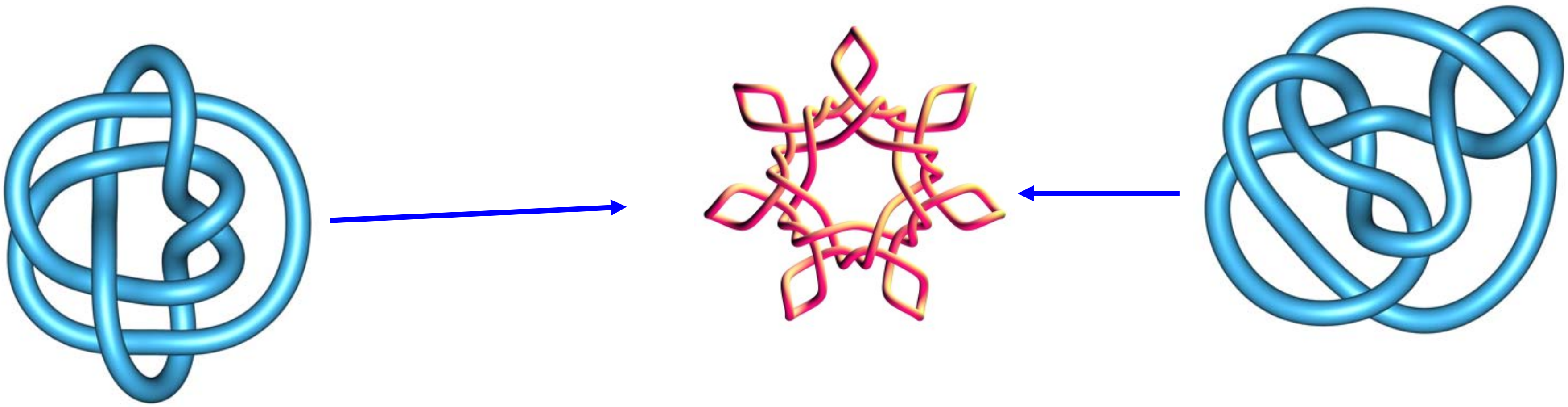
BOR**MEME**: used on 15 covers or articles!

Pictures are more democratic but they come from formulae

When is a Movie an Interactive Proof?

The Perko Pair 10_{161} and 10_{162}

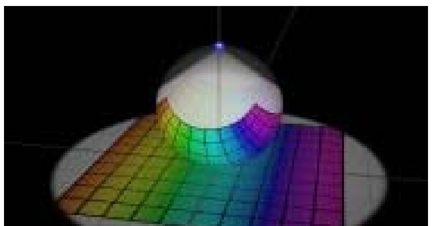
are two adjacent 10-crossing knots (1900)

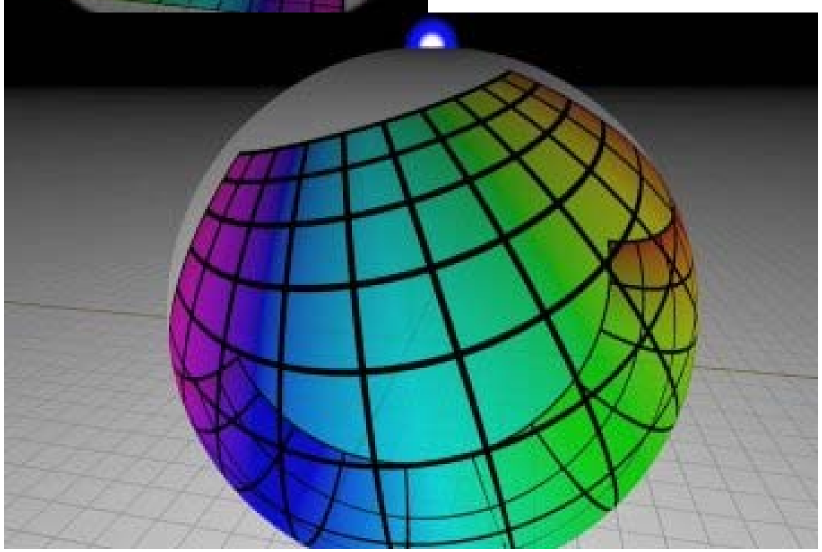


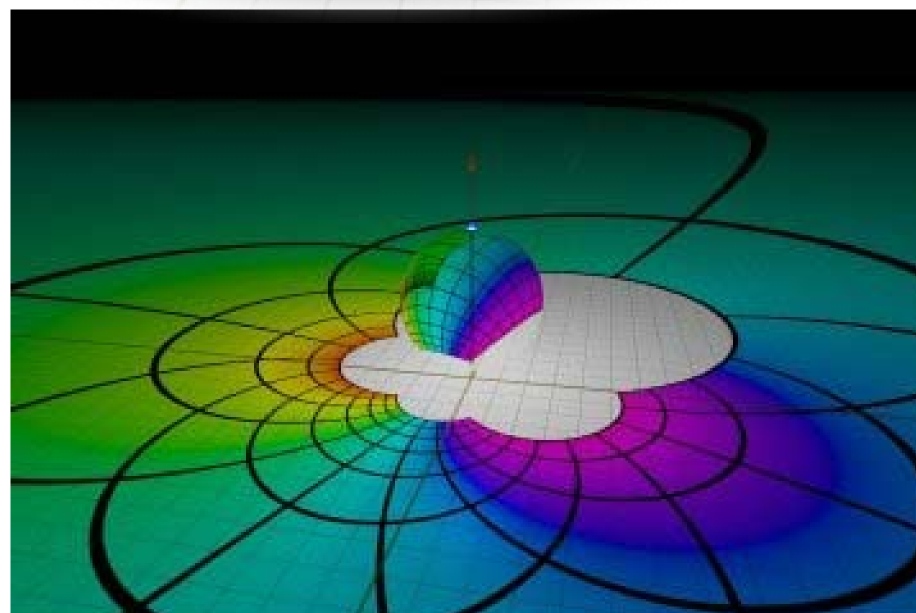
- **first shown to be the same by Ken Perko in 1974**
- **and beautifully made dynamic in KnotPlot (open source-ish)**
    - I'll happily play this movie and the next one if asked in the Question Period

A Movie that Teaches Beautifully

- Arnold and Rogness (2007)







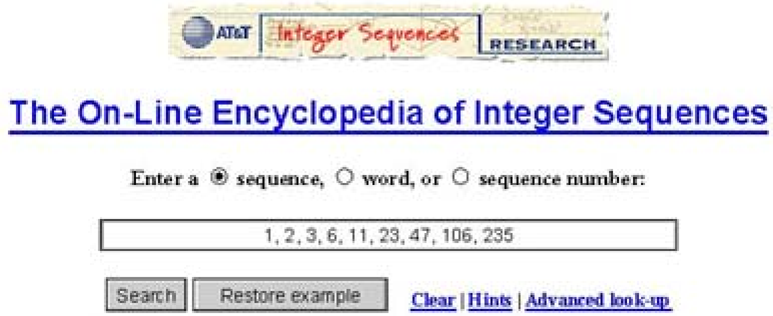
www.ams.org/notices/200810/tx081001226p.pdf

"What it comes down to is our software is too hard and our hardware is too soft."

Amazing New Web Services

- **AT&T** Online Encyclopedia of Sequences

  What is 1,2,3,6,11,23,47,106,235,…?



Supernumerary Rainbow over Newton's birthplace

- **NIST** Digital Library of Math Functions

  What is an Airy Function?

- **MAA** Digital Library with my company's free dictionary
  - also in *Maple* since 9.5

Soon the texts will also do lots of the maths

Greetings from the On-Line Encyclopedia of Integer Sequences!

Matches (up to a limit of 30) found for 1 2 3 6 11 23 47 106 235 :
[It may take a few minutes to search the whole database, depending on how many matches are found (the second and later look are faster)]

An Exemplary Database

ID Number: A000055 (Formerly M0791 and N0299)
URL: http://www.research.att.com/projects/OEIS?Anum=A000055
Sequence: 1,1,1,1,2,3,6,11,23,47,106,235,551,1301,3159,7741,19320, 48629,123867,317955,823065,2144505,5623756,14828074, 39299897,104636890,279793450,751065460,2023443032, 5469566585,14830871802,40330829030,109972410221
Name: Number of trees with n unlabeled nodes.
Comments: Also, number of unlabeled 2-gonal 2-trees with n 2-gons.
References
F. Bergeron, G. Labelle and P. Leroux, Combinatorial Species and Tree-Like Structures, Camb. 1998, p. 279.
N. L. Biggs et al., Graph Theory 1736-1936, Oxford, 1976, p. 49.
S. R. Finch, Mathematical Constants, Cambridge, 2003, pp. 295-316.
D. D. Grant, The stability index of graphs, pp. 29-52 of Combinatorial Mathematics (Proceedings 2nd Australian Conf.), Lect. Notes Math. 403, 1974.
F. Harary, Graph Theory. Addison-Wesley, Reading, MA, 1969, p. 232.
F. Harary and E. M. Palmer, Graphical Enumeration, Academic Press, NY, 1973, p. 58 and 244.
D. E. Knuth, Fundamental Algorithms, 3d Ed. 1997, pp. 386-88.
R. C. Read and R. J. Wilson, An Atlas of Graphs, Oxford, 1998.
J. Riordan, An Introduction to Combinatorial Analysis, Wiley, 1958, p. 138.
Links:
P. J. Cameron, Sequences realized by oligomorphic permutation groups J. Integ. Seqs. Vo
Steven Finch, Otter's Tree Enumeration Constants
E. M. Rains and N. J. A. Sloane, On Cayley's Enumeration of Alkanes (or 4-Valent Trees),
N. J. A. Sloane, Illustration of initial terms
E. W. Weisstein, Link to a section of The World of Mathematics.
Index entries for sequences related to trees
Index entries for "core" sequences
G. Labelle, C. Lamathe and P. Leroux, Labeled and unlabeled enumeration of k-gonal 2-tr
Formula: G.f.: A(x) = 1 + T(x)-T^2(x)/2+T(x^2)/2, where T(x) = x + x^2 + 2*x^3 + ...

Integrated real time use

- moderated
- 142,759 entries
- grows daily
- AP book had 5,000

http://dlmf.nist.gov First 21C database

Digital Library of Mathematical Functions

Chapter AI. Airy & Related Functions

Properties

- Index
- Notation
- Search

for what [Go]

Show Annotations

Need Help?

About the Project

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DLMF_feedback@nist.gov

January 12, 2006

Supported by NIST National Institute of Standards and Technology and National Science Foundation Celebrating 50 Years

§AI.3. Graphs & Visualizations | §AI.5. Integral Representations

§AI.4. Maclaurin Series

For $z \in \mathbb{C}$

AI.4.1

$$\mathrm{Ai}(z) = \mathrm{Ai}(0)\left(1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\cdots\right)+\mathrm{Ai}'(0)\left(z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\cdots\right)$$

Symbols used:
AiryAi, cdots and z

A&S Ref:
10.4.2 (with 10.4.4 and 10.4.5)

Encodings:
LaTeX

Parsed:

```
AiryAi@(z) = AiryAi@(0) * (1 + (1 / 3!) * z ^ 3 + ((1 cdot 4) / 6!) * z ^ 6 + ((1 cdot 4 cdot 7) / 9!) * z ^ 9 + cdots) + (diffop@(AiryAi, 1))@(0) * (z + (2 / 4!) * z ^ 4 + ((2 cdot 5) / 7!) * z ^ 7 + ((2 cdot 5 cdot 8) / 10!) * z ^ 10 + cdots)
```

- **Formula level metadata**
- **Mathematical searching**
- **Accessible output**
- **Latex, PNG, MathML**

AI.4.2

$$\mathrm{Ai}'(z) = \mathrm{Ai}'(0)\left(1+\frac{2}{3!}z^3+\frac{2\cdot 5}{6!}z^6+\frac{2\cdot 5\cdot 8}{9!}z^9+\cdots\right)+\mathrm{Ai}(0)\left(\frac{1}{2!}z^2+\frac{1\cdot 4}{5!}z^5+\frac{1\cdot 4\cdot 7}{8!}z^8+\cdots\right),$$

AI.4.3

$$\mathrm{Bi}(z) = \mathrm{Bi}(0)\left(1+\frac{1}{3!}z^3+\frac{1\cdot 4}{6!}z^6+\frac{1\cdot 4\cdot 7}{9!}z^9+\cdots\right)+\mathrm{Bi}'(0)\left(z+\frac{2}{4!}z^4+\frac{2\cdot 5}{7!}z^7+\frac{2\cdot 5\cdot 8}{10!}z^{10}+\cdots\right)$$

AI.4.4

$$\mathrm{Bi}'(z) = \mathrm{Bi}'(0)\left(1+\frac{2}{3!}z^3+\frac{2\cdot 5}{6!}z^6+\frac{2\cdot 5\cdot 8}{9!}z^9+\cdots\right)+\mathrm{Bi}(0)\left(\frac{1}{2!}z^2+\frac{1\cdot 4}{5!}z^5+\frac{1\cdot 4\cdot 7}{8!}z^8+\cdots\right).$$

§AI.3. Graphs & Visualizations | §AI.5. Integral Representations

The faint line below the main colored arc is a `supernumerary rainbow', produced by the interference of different sun-rays traversing a raindrop and emerging in the same direction. For each color, the intensity profile across the rainbow is an Airy function. Airy invented his function in 1838 precisely to describe this phenomenon more accurately than Young had done in 1800 when pointing out that supernumerary rainbows require the wave theory of light and are impossible to explain with Newton's picture of light as a stream of independent corpuscles. The house in the picture is Newton's birthplace.

CON

$x_i)\}\,\mathrm{d}t$

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- Ind
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N=6 onal on

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