Computer-Assisted Research Mathematics and its Applications

Functional and Nonlinear Analysis Workshop

2 – 4 October 2010

Abstracts

http://carma.newcastle.edu.au/

Fixed point property for Banach algebras associated to locally compact groups

Anthony To-Ming Lau
University of Alberta

Let E be a Banach space and C be a non-empty closed convex subset of E. A mapping T from C into C is non-expansive if $kT(x) - T(y)k \ \leq \ kx - yk$ for all $x, y \ \in C$. E is said to have the weak fixed point property if, for any non-empty weakly compact convex subset C of E, and non-expansive mapping $T\colon C \to C$, there is an $x \in C$ such that $T(x) = x$. Felix Browder (1965) shows that every uniformly convex Banach space has the weak fixed point property.

In this talk, I shall discuss the weak (resp. weak*) fixed point property for certain (resp. dual) Banach spaces associated to a locally compact group G and its relation with Radon Nikodym Property and continuous unitary representation of G.

Grothendieck and Vector Measures

Joe Diestel
Kent State University

Grothendieck's contributions to Banach space theory are many and now recognized properly. His contributions to the theory of vector-valued measures are not always recognized though they are many.

We will talk about three aspects of Grothendieck's efforts that are often overlooked as being due to him. First, we will take a look at his view of measures having finite variation. This is tied up in his beautiful theory of integral operators and ties together with the theory of differentiation with one of his most surprising results.

Next we'll discuss his work on operators on $C(K)$-spaces, especially his isolation of isomorphic invariants, like the Dunford-Pettis property.

Finally, a look at some of his most overlooked contributions coming, as they do as exercises in his still-wonderful text on topological vector spaces.

Douglas-Ratchford iterations in the absence of convexity.

Jonathan M Borwein and Brailey Sims
The University of Newcastle

The Douglas-Rachford iteration scheme, introduced half a century ago in connection with nonlinear heat flow problems, aims to find a point common to two or more closed constraint sets. Convergence is ensured when the sets are convex subsets of a Hilbert space, however, despite the absence of satisfactory theoretical justification, the scheme has been routinely used to successfully solve a diversity of practical optimization or feasibility problems in which one or more of the constraints involved is non-convex. As a first step toward addressing this deficiency, we provide convergence results for a proto-typical non-convex scenario. This is joint work with Brailey Sims. It is to appear in *Fixed-Point Algorithms for Inverse Problems in Science and Engineering* in the *Springer Optimization and Its Applications* series.
It is available at http://www.carma.newcastle.edu.au/jon/dr.pdf.

An elementary proof of James' characterization of weak compactness

Warren Moors
University of Auckland

In this talk I we provide an elementary proof of James' characterization of weak compactness in separable Banach spaces.

The proof of the theorem does not rely upon either Simon's inequality or any integral representation theorems. In fact, the proof only requires the Krein-Milman theorem, Milman's theorem and the Bishop-Phelps theorem.

Some recent fixed point results for asymptotic pointwise mappings

Wojciech M. Kozlowski
University of New South Wales

In 2008, Kirk and Xu proved the existence of fixed points of asymptotic pointwise nonexpansive mappings $T: C \to C$, i.e., mappings such that

$$|T^n(x) - T^n(y)| \leq \alpha_n(x)|x - y|$$

where $\limsup_{n\to\infty} \alpha_n(x) \leq 1$, for all $x, y \in C$, where C is a nonempty, closed, bounded and convex subset of a uniformly convex Banach space X. We will present recent results by Kozlowski on the weak and strong convergence of some iterative algorithms for the construction of the fixed points of the asymptotic pointwise nonexpansive mappings. Also, we will present results of the joint 2010 research of Khamsi and Kozlowski on fixed point theorems for asymptotic pointwise contractions and asymptotic pointwise nonexpansive mappings acting in modular functions spaces. The class of modular function spaces include both function and sequence variants of many classical spaces, like Lebesgue, Köthe, Orlicz, Musielak-Orlicz, Lorentz, Orlicz-Lorentz, Calderon-Lozanovskii. The mappings under considerations are asymptotic pointwise nonexpansive (or contractive) in the modular, and not necessarily in the norm sense, which is important from the perspective of applications.

Sampling in PSI spaces and a naive wavelet construction

Jeff Hogan
The University of Newcastle

In this talk we present some results from the theory of sampling in PSI (Principal Shift-Invariant) spaces – the building blocks of wavelet analysis. We review several sampling schemes and determine conditions under which they are valid. The main difficulty in working in these spaces – their non-translation-invariance – will be addressed. A by-product of this research has been a novel wavelet construction in which desirable properties of scaling functions may be obtained by fixing their values at the integers and using this data as the starting point of the construction.

Asplund operators and the Szlenk index

Phillip Brooker
PhD Student, Australian National University

The Szlenk index is an isomorphic invariant of a Banach space that can be thought of as a way of measuring the 'Asplundness' of a Banach space. It has arisen in a number of different areas of Banach space theory, for example the study of universality problems, the geometry of $C(K)$ spaces, metric geometry of separable spaces and, quite recently, fixed-point theory. In this talk, we discuss results from the author's recent PhD thesis on the Szlenk index of a Banach space and its natural operator analogue. The starting point of these results is a theorem asserting the existence of a class of closed operator ideals naturally associated with the Szlenk index. We shall discuss the relationship between these operator ideals and the well-known closed operator ideals of Asplund operators and separable range operators. We also consider the question of whether the operator ideals associated with the Szlenk index possess the factorisation property; the results presented in this direction are a refinement of the independent efforts of Reinov, Heinrich and Stegall in the late 1970s/early 1980s showing that every Asplund operator factors through an Asplund space. Our methods include an analysis of the behaviour of the Szlenk index under interpolative methods and the process of taking direct sums (over infinite index sets).

Analysis for Radical Banach Algebras

George Willis
The University of Newcastle

The *Gelfand transform* unifies many important ideas in analysis, including the Fourier and Laplace transforms and the spectral analysis of linear operators. It represents an abstract commutative Banach algebra as an algebra of continuous functions on a topological space, called the *carrier space* of the algebra. However the Gelfand transform can fail to be faithful, and indeed in some cases the entire algebra maps to the zero function. Such algebras, which are called *radical*, arise for example from the failure of spectral synthesis in harmonic analysis or from linear operators with zero spectrum.

This talk discusses the notion of *just integral domain* and proposes that this idea could be used to analyse general radical Banach algebras. Inspiration for this concept comes from group theory and the concept of *just infinite* group. Conjectures about just integral domains will be presented that are suggested by results about just infinite groups.

Quasi-Banach spaces, M-ideals and polyhedra

David Yost
University of Ballarat

Three-space problems and extensions of Banach spaces lead naturally to the study of quasi-linear maps between quasi-Banach spaces. We define a more restricted class of mappings which keeps us within the class of Banach spaces. More precisely, a mapping between Banach spaces is called pseudolinear if it is homogeneous and its linearity gap is bounded by the gap in the triangle inequality. The twisted sums which arise then turn out to be semi-L-summands. Applying the dual concept of semi-M-ideals leads us to some interesting results about best approximant sets and Minkowski decomposability of balls in certain Banach spaces. The special case of finite dimensional spaces whose unit balls are polytopes then becomes a theorem about polytopes. Once stated, this result can be proved geometrically, but its inadvertent discovery required a lot of functional analysis.

Enhanced negative type for finite metric trees

Ian Doust
University of New South Wales

The p-negative type and generalized roundness inequalities arose classically in studies of isometric embeddings: when does a metric space embed isometrically in a Hilbert space or in an L^p space? These ideas remain of great interest in areas ranging from functional analysis to theoretical computer science. Hjorth *et al*. have shown that finite metric trees have strict 1-negative type. In this talk I shall describe joint work with Tony Weston in which we show that a new and substantially stronger family of geometric inequalities holds for finite metric trees.