Number Theory Online Conference 2020 Schedule and Abstracts

3 – 5 June 2020

1 Schedule

Time AEST	**Wednesday 3 June**	
10h00-10h15	Welcome	
10h30-11h00	Felipe Voloch	*Sha versus the Volcano*
11h30-12h00	Bo-Hae Im	*Waring's problem for rational functions in one variable*
12h30-13h00	Igor Shparlinski	*Denominators of rational numbers in or close to the Cantor set*
14h00-14h30	Tim Trudgian	*Zeta zeroes: mind the gap!*
15h00-15h30	Thomas Morrill	*Look, Knave*

Time AEST	**Thursday 4 June**	
10h00-10h50	Julie Wang	*On Pisot's d-th root conjecture for function fields and related GCD estimates*
11h00-11h30	Jon Grantham	*Binary and Ternary Curves of Fixed Genus and Gonality with Many Points*
12h00-12h30	Elisa Bellah	*Norm Form Equations and Linear Divisibility Sequences*
14h00-14h30	Matteo Bordignon	*Partial Gaussian sums and Pólya–Vinogradov inequality for primitive characters*
15h00-16h00	Group Discussion	*The Future of Online Conferences*

Time AEST	**Friday 5 June**	
10h00-10h50	Daniel Delbourgo	*Variation of Iwasawa invariants over number fields*
11h00-11h30	Fu-Tsun Wei	*On class number relations and intersections over function fields*
11h30-11h45	Group Photo	in iSee
12h00-12h30	Peng-Jie Wong	*Refinements of Strong Multiplicity One for GL(2)*
13h00-13h30	Mumtaz Hussain	*Uniform Diophantine approximation: improving Dirichlet's theorem*

2 Useful Links

- Conference website: `https://carma.newcastle.edu.au/meetings/ntoc2020`
- For help, email `carma@newcastle.edu.au`

3 Abstracts

3.1 Norm Form Equations and Linear Divisibility Sequences

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Finding integer solutions to norm form equations is a classic Diophantine problem. Using the units of the associated coefficient ring, we can produce sequences of solutions to these equations. It turns out that such a sequence can be written as a tuple of integer linear recurrence sequences, each with characteristic polynomial equal to the minimal polynomial of our unit. We show that in some cases, these sequences are linear divisibility sequences.

3.2 Partial Gaussian sums and Pólya–Vinogradov inequality for primitive characters

Matteo Bordignon
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We will improve, for large moduli, on the best explicit version of the Pólya–Vinogradov inequality for primitive characters, due to Frolenkov and Soudararajan, drawing inspiration from a paper by Hildebrand.

3.3 Variation of Iwasawa invariants over number fields

Daniel Delbourgo
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Iwasawa theory provides an intriguing p-adic link between the algebraic world and the analytic world. A key quantity is the lambda-invariant, which counts the number of zeroes of the p-adic L-function. We shall carefully explain how this invariant (both the algebraic and analytic version) behaves over a non-abelian extension $K/\mathbb{Q}$, as one moves around a Hida family of modular forms.

3.4 Binary and Ternary Curves of Fixed Genus and Gonality with Many Points

Jon Grantham, Xander Faber
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We determine the maximum number of rational points on curves over $\mathbb{F}_2$ and $\mathbb{F}_3$ with fixed gonality and small genus.

3.5 Uniform Diophantine approximation: improving Dirichlet's theorem

Mumtaz Hussain
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In this talk, I will discuss the metrical theory associated with the set of Dirichlet non-improvable numbers.

Let $\Psi : [1,\infty) \to \mathbb{R}_+$ be a non-decreasing function, $a_n(x)$ the n'th partial quotient of x and $q_n(x)$ the denominator of the n'th convergent. The set of Ψ-Dirichlet non-improvable numbers

$$G(\Psi) := \Big\{x \in [0,1) : a_n(x)a_{n+1}(x) \ > \ \Psi\big(q_n(x)\big) \text{ for infinitely many } n \in \mathbb{N}\Big\},$$

is related with the classical set of $1/q^2\Psi(q)$-approximable numbers

$$\mathcal{K}(\Psi) := \left\{x \in [0,1) : \left|x - \frac{p}{q}\right| < \frac{1}{q^2\Psi(q)} \text{ for infinitely many } (p,q) \in \mathbb{Z} \times \mathbb{N}\right\},$$

in the sense that $\mathcal{K}(3\Psi) \subset G(\Psi)$. In this talk, I will explain that the Hausdorff measure of the set $G(\Psi)$ obeys a zero-infinity law for a large class of dimension functions. Together with the Lebesgue measure-theoretic results established by Kleinbock & Wadleigh (2016), our results contribute to building a complete metric theory for the set of Dirichlet non-improvable numbers.

Another recent result that I will discuss will be the Hausdorff dimension of the set $G(\Psi) \setminus \mathcal{K}(3\Psi)$.

3.6 Waring's problem for rational functions in one variable

Bo-Hae Im, Michael Larsen
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Let $f \in \mathbb{Q}(x)$ be a non-constant rational function. We consider "Waring's Problem for $f(x)$," i.e., whether every element of $\mathbb{Q}$ can be written as a bounded sum of elements of $\{f(a) \mid a \in \mathbb{Q}\}$. For rational functions of degree 2, we give necessary and sufficient conditions. For higher degrees, we prove that every polynomial of odd degree and every odd Laurent polynomial satisfies Waring's Problem. We also consider the "Easier Waring's Problem': whether every element of $\mathbb{Q}$ can be represented as a bounded sum of elements of $\{\pm f(a) \mid a \in \mathbb{Q}\}$. This is a joint work with Michael Larsen.

3.7 Look, Knave

Thomas Morrill
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In this lighthearted tribute to the late John Conway, we examine a recursive sequence in which s_n is a binary string that describes what the previous term s_{n-1} is not. By adapting Conway's approach on the Look-and-Say sequence, we determine limiting behaviour of $\{s_n\}$ and dynamics of a related self-map on the space of infinite binary sequences. Our main result is the existence and uniqueness of a pair of binary sequences, each the compliment-description of the other.

3.8 Denominators of rational numbers in or close to the Cantor set

Igor Shparlinski
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We start with a short survey of recent results on the arithmetic structure of denominators of rational numbers which belong to the Cantor set, or are very close to one of each elements. We then describe how a new point of approcah, using classical bounds of short exponential sums due to N. Korobov, leads to new results and improvements of some existing results.

3.9 Zeta zeroes: mind the gap!

Tim Trudgian, Aleks Simonic and Caroline Turnage-Butterbaugh
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I shall present some hot-off-the-press work, with AS and CTB, on gaps between the zeroes of the Riemann zeta-function. I recorded this talk with a document camera, and will give commentary as "Current Tim" over the writings of "Past Tim" — nothing could go wrong!

3.10 Sha versus the Volcano

Felipe Voloch, Brendan Creutz
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We describe the structure of the Tate-Shafarevich group of constant elliptic curves over function fields by exploiting the volcano structure of isogeny graphs of elliptic curves over finite fields.

3.11 On Pisot's d-th root conjecture for function fields and related GCD estimates

Julie Tzu-Yueh Wang, Ji Guo, Chia-Liang Sun
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Let $B(X) = \sum_{n=0}^{\infty} b(n)X^n$ represent a rational function in $\mathbb{Q}(X)$ and suppose that $b(n)$ is a perfect d-th power for all large $n \in \mathbb{N}$. Pisot's d-th root conjecture states that one can choose a d-th root $a(n)$ of $b(n)$ such that $A(X) := \sum a(n)X^n$ is again a rational function. In this talk, we propose a function-field analog of Pisot's d-th root conjecture and prove it under some "non-triviality" assumption. We relate the problem to a result of Pasten-Wang on Buchi's d-th power problem

and develop a function-field analog of an GCD estimate in a recent work of Levin-Wang. This is a joint work with Ji Guo and Chia-Liang Sun.

3.12 On class number relations and intersections over function fields

Fu-Tsun Wei
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In this talk, we shall discuss a function field analogue of the Hirzebrush-Zagier class number formula. More precisely, we establish a connection between class numbers of "imaginary" quadratic function fields and the corresponding intersections of "Heegner-type" divisors on the Drinfeld-Stuhler modular surfaces. The main bridge is the theta series associated to anisotropic quadratic spaces of dimension 4. The connection directly comes from two different expressions of the Fourier coefficients of the theta series, which can be viewed as a geometric Siegel-Weil formula in this particular case. This is a joint work with Jia-Wei Guo.

3.13 Refinements of Strong Multiplicity One for GL(2)

Peng-Jie Wong
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Let f_1 and f_2 be holomorphic newforms of same weight and with same nebentypus, and let $a_{f_1}(n)$ and $a_{f_2}(n)$ denote the Fourier coefficients of f_1 and f_2, respectively. By the strong multiplicity one theorem, it is known that if $a_{f_1}(p) = a_{f_2}(p)$ for almost all primes p, then f_1 and f_2 are equivalent. Furthermore, a result of Ramakrishnan states that if $a_{f_1}(p)^2 = a_{f_2}(p)^2$ outside a set of primes p of density less than $\frac{1}{18}$, then f_1 and f_2 are twist-equivalent.

In this talk, we will discuss some refinements and variants of the strong multiplicity one theorem and Ramakrishnan's result for general $\mathrm{GL}(2)$-forms. In particular, we will analyse the set of primes p for which $|a_{f_1}(p)| \neq |a_{f_2}(p)|$ for non-twist-equivalent f_1 and f_2.

4 Organising Committee

David Allingham, Florian Breuer, Michael Coons, Thomas Morrill, Alina Ostafe and Juliane Turner.

5 Sponsors

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