# Theory and Applications of Convex and Non-convex Feasibility Problems

Laureate Prof. Jonathan Borwein with Matthew Tam
http://carma.newcastle.edu.au/DRmethods/paseky.html

THE UNIVERSITY OF NEWCASTLE AUSTRALIA

CARMA

Spring School on Variational Analysis VI
Paseky nad Jizerou, April 19–25, 2015

Last Revised: May 6, 2016

# Spring School on Variational Analysis 2015

For Spring School on Function Spaces and Lineability 2015, click here

*What am I if I will not participate?*
*- Antoine de Saint-Exupery*

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**Dear Colleague,**

Following a longstanding tradition, the Faculty of Mathematics and Physics, Charles University in Prague and the Academy of Sciences of the Czech Republic will organize the Spring School on Variational Analysis VI. The School will be held in Paseky nad Jizerou, in a chalet in the Krkonose Mountains, **April 19 - 25, 2015**.

The program will consist of series of lectures on

## Variational Analysis

### and its Applications

delivered by

| |
|---|
| Jonathan M. Borwein<br>The University of Newcastle, Australia<br>Theory and Applications of Convex and Non-convex Feasibility Problems |
| Marián Fabian<br>Academy of Sciences of the Czech Republic, Prague, Czech Republic<br>Separable Reductions and Rich Families in Theory of Fréchet Subdifferentials |
| Alexander Ioffe<br>Technion, Haifa, Israel<br>Variational Analysis and Optimization Theory |
| David Russell Luke<br>Georg-August-Universität Göttingen, Germany<br>Variational Methods in Numerical Analysis |

The purpose of this meeting is to bring together researchers with common interest in the field. There will be opportunities for informal discussions. Graduate students and others beginning their mathematical career are encouraged to participate.

# Introduction

A feasibility problem requests solution to the problem

$$\text{Find} \quad x \in \bigcap_{i=1}^{N} C_i,$$

where $C_1, C_2, \ldots C_N$ are closed sets lying in a Hilbert space $\mathcal{H}$.

We consider iterative methods based on the non-expansive properties of the metric projection operator

$$P_C(x) := \mathrm{argmin}_{c \in C} \|x - c\|$$

or reflection operator $R_C := 2P_C - I$ on a closed convex set $C$.

The two methods which we focus are on the method of alternating projections (MAP) and the Douglas–Rachford method (DR).

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# Introduction

These methods work best when the projection on each set $C_i$ is easy to describe or approximate. These methods are especially useful when the number of sets involved is large as the methods are fairly easy to parallelize.

The theory is pretty well understood when all sets are convex but much less clear in the non-convex case. But as we shall see application of this case has had may successes. So this is a fertile area for both pure and applied study.

The five hours of lectures will cover the following topics.

1. **Feasibility problems:** convex theory, nonexpansivity, Fejér monotonicity & convergence of MAP and variants.
2. **The Douglas–Rachford Method:** convex Douglas–Rachford iterations and variants.
3. **Non-convex Douglas Rachford iterations** and **iterative geometry**.
4. **Applications to completion problems**: an introduction & detailed case studies.

- Each lecture will contain closing commentary, open questions, and exercises.

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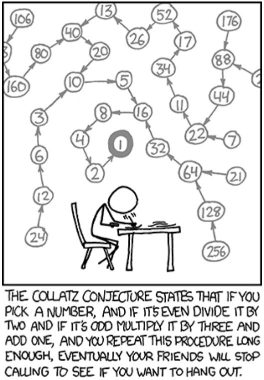
# Motivation

The need to integrate and iterate real theory with real models for real applications:

- Good theoretical understanding
  - you can not use what you do not know
  - you can work inductively
- Careful modelling of applications
  - the model matters especially in the nonconvex case
  - moving to application specific refinements
- Good implementations
  - starting with ‘general purpose agents’
  - moving to application specific refinements

# Introduction


Lectures are available online at:

http://carma.newcastle.edu.au/DRmethods/paseky.html